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## DYNAMICS OF RELATIVE TORSIONAL VIBRATIONS IN THE FORMATION OF A REGULAR MICRORELIEF ON INTERNAL CYLINDRICAL SURFACES

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**Summary.** The article presents the results of analysis of modern literature sources in search of mathematical models describing the dynamics of the process of forming regular microrelief on the inner cylindrical surfaces of parts operating in difficult conditions, in order to increase their life cycle. The absence of mathematical models describing this process and the peculiarities of its implementation with the point action of the deforming element on the surface of the body part are established. The movements of the tool during the process of forming a regular microrelief on the inner cylindrical surface of the body of the part are considered and the driving forces that follow this process are analyzed. Based on the results of the analysis, a mathematical model of the dynamic process of forming regular microrelief on the inner cylindrical surface of the body of the part was developed. The peculiarity of this process is that microrelief is formed by concentrated force, the point of application of which is constantly changing in the radial and axial directions relative to the part. Therefore, the mathematical model that describes this process will have a discrete right-hand side. It is proposed to model such an action using Dirac delta functions with linear and temporal variables, using the method of regularization of these features, in particular, existing methods of integrating the corresponding nonlinear mathematical models of torsional vibrations of a part. Analytical relations describing these vibrations in the process of forming a regular microrelief are obtained. Using Maple software 3D changes in torsion angle depending on different values of the source data are constructed. The conducted research will allow to consider torsional oscillations that is crucial for long cylindrical details, such as sleeves of hydraulic cylinders, parts of drilling mechanisms and others.

**Key words:** technology, cylindrical surface, quality parameters, vibration processing, torsional vibrations, mathematical model.

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**Statement of the problem.** Improving the performance of the working surfaces of machine parts used in oil and gas industry, especially in those shaped as rotation bodies is an important task of machine-building production. Operating conditions of such parts are characterized by high working temperatures and specific pressures, which leads to adhesive wear, setting and rapid destruction of their working surfaces. One of the progressive ways to improve the performance of the parts surfaces made in the form of rotation bodies are the methods of surface plastic deformation, in particular balls or rollers treatment. Such treatment can significantly reduce surface roughness, increase the surface microhardness and, in some cases, avoid the need for quenching [1, 2, 3].

**Analysis of the available investigations.** A separate area in this field are the methods of dynamic action on the treated surface. Their essence is a high-intensity periodic action on the treated surface with an indenter, which is usually ball-shaped. Dynamic action ensures less deformation force, provides a larger bearing surface area, and, accordingly, better performance, which are described by the Abbott-Firestone curve. In [4], the advantages of the method of dynamic action with the construction of the above-mentioned surface characteristics are presented.

It was suggested to estimate the operational properties of the surface according to the parameters of this curve by other scientists. In particular, [5, 6] describe a method where the evaluation of the performance properties of the surface is carried out not by the surface roughness, but using the parameters of the Abbott-Firestone curve. The authors state that the three parameters determined from the Abbott-Firestone curve  $R_{pk}$ ,  $R_k$  and  $R_{vk}$  characterize the ability of the surface to resist frictional wear.

The dynamic action on the treated surface can be chaotic or ordered with the formation of an ordered microrelief on it.

The founder of application of ways and methods of forming regular microreliefs was Schneider Yu.G. In his work [7], the ways and methods of forming a regular microrelief, tool design and processing modes were described. He also proposed a classification of regular microreliefs formed on flat and cylindrical surfaces with different geometric parameters.

The results of his research became the basis for creation of the standard GOST 24773-81 [8], which regulates the parameters of regular microrelief formed on flat and cylindrical surfaces.

The development of the basics of formation of regular microreliefs is given in [9], where the advantages of surfaces with formed microreliefs over surfaces treated by other methods are described.

High-performance tool complexes, their structure and principles of work are described in [10, 11]. Such complexes provide formation of microreliefs on profile surfaces of any complexity.

In [12], the author for the first time presents mathematical models that describe regular microreliefs on the end surfaces of bodies rotation, classifies such microreliefs and mathematically describes their parameters.

In [13], the scheme, technological equipment and tools for formation of regular microrelief on flat and spherical surfaces using pressures of 40 MPa on a 5-coordinate milling machine with digital program control, the microstructure of the surface with formed microrelief is studied, two approaches to formation of regular microreliefs on complex profile surfaces are compared.

Geometric dimensions of the elements of the regular microrelief are rather small (only 1–3 mm), so the influence of any factors can distort it. This is especially true of the parameters of its regularity, which provide stable physical and mechanical properties of the working surfaces of machine parts.

In [14], the results of theoretical research on the dynamic characteristics of the method of vibration-centrifugal hardening of long-dimensional metal cylindrical parts are presented. The research method is described, the spatial schematic diagram of the vibration-centrifugal reinforcing tool with electromagnetic drive and elastic systems is given. On the basis of researches, empirical dependences for determining the basic dynamic characteristics of a method are obtained and analyzed. An algorithm for analyzing the dynamic characteristics of the contact interaction of the working bodies of the electromagnetic hardener with elastic systems and the treated surface is developed.

Based on the analysis, it can be stated that in modern research much attention is paid to methods and means of forming a regular microrelief on the inner cylindrical surfaces as a way to ensure the required quality of these types of surfaces. However, the issues of mathematical modeling of the processes of formation of regular microrelief on the inner cylindrical surfaces need further study.

**The objective of the paper** is to develop a mathematical model that describes the relative torsional vibrations of a body during the formation of regular microrelief on its inner cylindrical surface.

**Statement of the task.** To ensure the regularity of the microrelief, it is necessary to develop a dynamic model of its formation taking into account such parameters as physical and

mechanical properties of the treated surface, the magnitude of external action on the surface by the tool (vibrator), the nature of contact and more.

**Main material statement.** In this article we consider the formation of a regular microrelief on the inner cylindrical surface of the part. All the formative movements that follow the process of formation of a regular microrelief are shown in Fig. 1, these are: translation motion  $D_n$ , feed motion  $D_s$ , and reciprocating oscillating motion  $D_i$ .

The process of forming a regular microrelief on the inner cylindrical surface has its own characteristics, in particular:

- cylindrical treated surface rotates around a fixed axis;
- external action on the treated surface from the side of the body applying microrelief has a point character, in addition, the point of contact of these bodies changes in longitudinal and radial directions.

The components of these displacements and the angular velocity of rotation of the cylindrical surface affect the dynamic processes in the elastic cylindrical body that take place during the application of microrelief and determine the shape of the latter. As for the component of microrelief, which is due to dynamic processes in a cylindrical body, it is primarily caused by its longitudinal and torsional vibrations, and therefore determined by the elastic properties of the body, boundary conditions and external action.

From the above it follows that under the action of external force factors the elastic cylindrical body on which the microrelief is applied is in a complex motion [15, 16]. For this reason, to describe its dynamics, the article considers its following components:

- translation motion around the horizontal axis as an absolutely rigid body;
- relative motion:
  - a) torsional vibrations around the horizontal axis of the elastic body;
  - b) longitudinal vibrations of the elastic body.

These issues are the subject of research.

The main assumptions that underlie the description of the translation motion of the body (Fig. 1) are as follows:

- the body on the inner surface of which the microrelief is applied is hollow and homogeneous one and is of cylindrical shape with the following parameters: the outer radius is  $R_d$ , the inner radius is  $r_d$ , the length is  $l_d$ , the mass is  $M_d$ ;

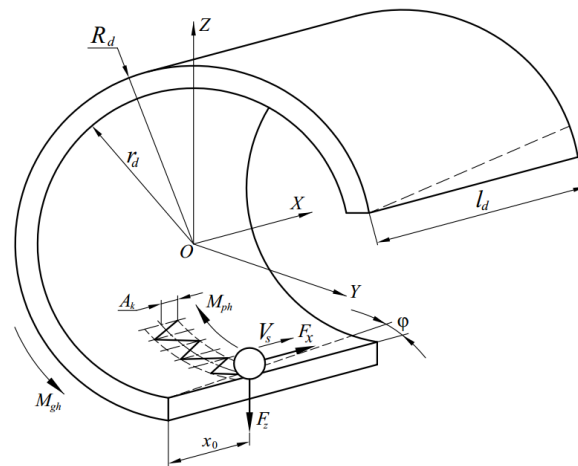
- active and passive forces acting on the treated body are:

a) clamping force from the point action of vibration roller, which applies microrelief to the inner cylindrical surface. This force has longitudinal (tangent)  $F_x$  and transverse (normal)  $F_z$  components (the axis  $OX$  is horizontal and coincides with the rotation axis, the axis  $OZ$  is vertical, the axis  $OY$  is horizontal);

b) the driving moment  $M_{gh}$ , which causes the workpiece to rotate around the horizontal axis, the shear moment ( $M_{ph}$ ) from the action of the vibration roller on the inner surface, which depends on the following factors: clamping force, shape of the vibration roller, hardness of the material, etc. It is assumed below that the latter factors are taken into account by the coefficient  $f_{ph}$  ( $M_{ph} = F_z \cdot r_d \cdot f_{ph}$ ).

In the article, the relative torsional vibrations of an elastic cylindrical hollow body in the process of forming microrelief on its inner surface are studied. The peculiarity of these vibrations is that they occur under the action of a discrete external load, which constantly changes its point of application. All this creates certain difficulties in finding solutions to boundary value problems that describe these vibrations.

As it was stated [17], the external action on the inner surface of a cylindrical body in the process of forming a regular microrelief on its inner surface leads to complex vibrations of the workpiece. Torsional vibrations are due to the transverse component  $F_z$ , (clamping force), which is directed in a perpendicularly to the surface (Fig. 1).



**Figure 1.** Calculation scheme for constructing a dynamic model of relative torsional vibrations of the treated body in the process of forming a regular microrelief on its inner surface

On indicating  $\varphi(x, t)$  – the angle of torsion of the normal section of a cylindrical surface with coordinate  $x$  at any time  $t$ , the differential equation of relative torsional vibrations under the assumptions about elastic and dissipative forces is

$$\varphi_{tt} - \alpha \varphi = \varepsilon \left[ \hat{\eta} \varphi_t + \hat{\beta}(\varphi) \varphi \right] + F_z r f \delta(x - \bar{x}_0 - Vt), \quad (1)$$

where  $\alpha^2 = G/\rho$ ,  $M_{rh} = \varphi_x G J_p$ ,  $J_p$  are inertia moment of the cross-section of a cylindrical body,  $G$  is modulus of elasticity of the second kind,  $f_{ph}$  is a coefficient of shear resistance (microrelief formation),  $F_z$  is a normal component of the clamping force;  $\delta(\dots)$  is a delta function of the corresponding argument;  $\hat{\eta}$   $\hat{\beta}$  are coefficients that characterize the resistance force and the nonlinear component of the restoring force.

Notes.

1. The article assumes that the tangent component at the contact point of the vibration roller with the treated surface is proportional to the normal component with a coefficient of proportionality  $f_{ph}$ .

2. The treated body is isotropic, so the coefficient of proportionality in the shear force in longitudinal direction is the same as in the transverse one.

As for the boundary conditions, they are as follows

$$\varphi(x, t) = 0, \quad \varphi(x, t) = \frac{M}{GJ}, \quad (2)$$

where  $M_{rh}$  is a drive moment at the beginning of the treated body;  $M=f(t)$ . To study the dynamics of torsional vibrations in the process of forming microrelief to a cylindrical surface, first of all, it is necessary to find a solution of the boundary value problem (1), (2) of a nonlinear differential equation with partial derivatives under inhomogeneous boundary conditions. To find it by substituting variables according to

$$\varphi(x, t) = \Phi(x, t) + \Psi(x, t) \quad (3)$$

we reduce the problem with inhomogeneous boundary conditions to the problem with homogeneous boundary conditions [18]. Therefore, in (1),  $\Phi(x, t)$  is the solution of equation

$$\Phi_{xx}(x,t) = 0 \tag{4}$$

that satisfies the inhomogeneous boundary conditions that follow (2), (3), i.e.

$$\Phi(x,t)|_{x=0} = 0, \quad \Phi_x(x,t)|_{x=l} = \frac{M}{GJ_p}. \tag{5}$$

Then  $\Psi(x,t)$  function must be the solution of the equation

$$\Psi_{tt} - \alpha^2 \Psi_{xx} = -\Phi_{tt}(x,t) + \alpha^2 \Phi_{xx}(x,t) - F_z R_d f_{ph} \delta(x - \bar{x}_0 - Vt) + \varepsilon \hat{\eta}(\Phi_t(x,t) + \Psi_t(x,t)) + \varepsilon \hat{\beta}[\Phi_{xx}(x,t) + \Psi_{xx}(x,t)][\Phi_x(x,t) + \Psi_x(x,t)]^2 \tag{6}$$

and satisfy the homogenous boundary conditions

$$\Psi(x,t)|_{x=0} = 0, \quad \Psi_x(x,t)|_{x=l} = 0. \tag{7}$$

Finding the solution of equation (2) under boundary conditions (3) is not difficult

$$\Phi(x,t) = C_1(t)x + C_2(t), \tag{8}$$

where  $C_2(t) = 0 \Rightarrow C_1(t) = \frac{M}{GJ_p}$

Thus,  $\Phi(x,t) = \frac{M}{GJ_p}x$ , and to determine the function  $\Psi(x,t)$ , we have a linear inhomogeneous equation

$$\Psi_{tt} - \alpha^2 \Psi_{xx} = -\Phi_{tt}(x,t) + \alpha^2 \Phi_{xx}(x,t) - F_z R_d f_{ph} \delta(x - x_0 - V_s t) + \varepsilon \hat{\eta}(\Phi_t(x,t) + \Psi_t(x,t)) + \varepsilon \hat{\beta}_{xx}(x,t) \Psi_{xx}(x,t) \left[ \frac{M}{GJ_p} + \Psi_x(x,t) \right]^2 \tag{9}$$

and it has to satisfy the homogenous boundary conditions (7).

The structure of equation (9) is similar to equation, which describes the relative longitudinal vibrations. Similar to longitudinal vibrations, we use general ideas of perturbation methods adapted to a similar class of equations to analyze relative torsional vibrations  $\Psi(x,t) = \Psi_0(x,t) + \varepsilon \Psi_1(x,t)$ . According to them, first of all, we find the effect of external perturbation on the dynamics of forming microrelief, i.e. find the solution of the main part of the specified equation, the equation at  $\varepsilon = 0$

$$\Psi_{0tt} - \alpha^2 \Psi_{0xx} = -F_z R_d f_{ph} \delta(x - \bar{x}_0 - V_s t) \tag{10}$$

We solve it as

$$\Psi_0(x,t) = \sum_k T_k(t) \tilde{X}_k(x), \tag{11}$$

where the system of functions  $\{X_k(x)\}$  must satisfy the boundary conditions  $\tilde{X}(x)|_{x=0} = 0$ ,

$$X(x)|_{x=l} = 0. \text{ Such system of functions is } \{ \tilde{X}_k(x) \} = \left\{ \sin \frac{(2k+1)\pi}{2l_d} x \right\}.$$

As for the functions  $T_k(t)$ , obviously they, as follows from the basic equation, must satisfy the inhomogeneous equation

$$\ddot{T}_k(t) + \alpha^2 \frac{1}{P_k} \left( \frac{(2k+1)\pi}{2l_d} \right)^2 T_k(t) = -\frac{1}{P_k} \int_0^l F_z R_d f_{ph} \delta(x - x_0 - Vt) \tilde{X}_k(x) dx, \tag{12}$$

where  $P_k = \int_0^l \left[ \sin \frac{(2k+1)\pi}{2l_d} x \right]^2 dx = \frac{l_d}{2}$

According to the properties of the delta function [19], the integral in the right part of equation (8) takes values  $\int_0^l \sin \frac{(2k+1)\pi}{2l_d} x \delta(x - \bar{x}_0 - V_s t) dx = \sin \left( \frac{(2k+1)\pi}{2l_d} (\bar{x}_0 + V_s t) \right)$ , and accordingly the specified differential equation is transformed into the form

$$\ddot{T}_k(t) + \omega_k^2 T_k(t) = -\frac{1}{P_k} F_z R_d f_{ph} \sin \left( \frac{(2k+1)\pi}{2l_d} (\bar{x}_0 + V_s t) \right), \tag{13}$$

where  $\omega_k^2 = \alpha^2 \frac{1}{P_k} \left( \frac{(2k+1)\pi}{2l_d} \right)^2$

Thus, the representation of the discrete action of external load on the processed cylindrical surface using the delta function in combination with the method of partial sampling and for the case of relative torsional vibrations allows to solve the problem, because finding the solution of equation (9) is not difficult. Indeed, equation (13) is linear inhomogeneous, and therefore the general solution of the corresponding homogeneous one is  $T_{0k}(t) = T_0 \sin(\omega_k t + \vartheta_{0k})$ , and the corresponding partial solution of the inhomogeneous equation (9) can be represented as

$$\tilde{T}_k(t) = -\frac{1}{P_k} F_z R_d f_{ph} \frac{1}{\omega_k} \int_0^t \sin \omega_k (t - \tau) \sin \frac{(2k+1)\pi}{2l_d} (\bar{x}_0 + V_s \tau) d\tau. \tag{14}$$

Collectively, the obtained results allow us to describe the multifrequency dynamic process of relative torsional vibrations of a cylindrical body in the process of forming microirregularities on its inner surface as

$$\begin{aligned} \Psi_0(x, t) = & \frac{M}{GJ_p} x + \sum_k \sin \frac{(2k+1)\pi}{2l_d} x \times \\ & \times \left\{ T_{0k} \cos(\omega_k t + \vartheta_0) - \frac{1}{P_k} F_z R_d f_{ph} \frac{1}{\omega_k} \int_0^t \sin \omega_k (t - \tau) \sin \frac{(2k+1)\pi}{2l_d} (x - \bar{x}_0 - V_s \tau) d\tau \right\} \end{aligned} \tag{15}$$

As for the first approximation, i.e. the influence of nonlinear and dissipative forces, their influence is determined by the differential equation

$$\Psi_{1tt}(x, t) - \alpha^2 \Psi_{1xx}(x, t) = \left[ \eta \Psi_{0t}(x, t) \right] + \hat{\beta} \left( \Psi_{0x}(x, t) \right)^2 \Psi_{0xx}(x, t) \tag{16}$$

and its solution is similar to the solution of the equation of relative longitudinal vibrations with the difference that the boundary conditions, and hence the system of eigenfunctions is slightly different. The latter is not an obstacle to the presentation of the function  $\varphi(x, t)$  as

$$\varphi(x,t) = \int_0^t \sin\left(\frac{(2k+1)\pi}{2l_d}x\right) \times \left\{ \frac{1}{l_d} \bar{f}_{ph} r_d F_Z \left(\frac{2k+1}{2l_d}\alpha\right)^{-1} \int_0^t \sin\left(\frac{(2k+1)\pi(x_0 + V_s\tau)}{2l_d}\right) \sin\left(\frac{(2k+1)\pi\alpha(t-\tau)}{2l_d}\right) d\tau + \frac{2l_d\varepsilon}{(2k+1)\pi\alpha} \int_0^t \cos\left(\alpha\frac{k\pi}{l_d}(t-\tau)\right) \bar{\Theta}(\tau) d\tau \right\} \quad (17)$$

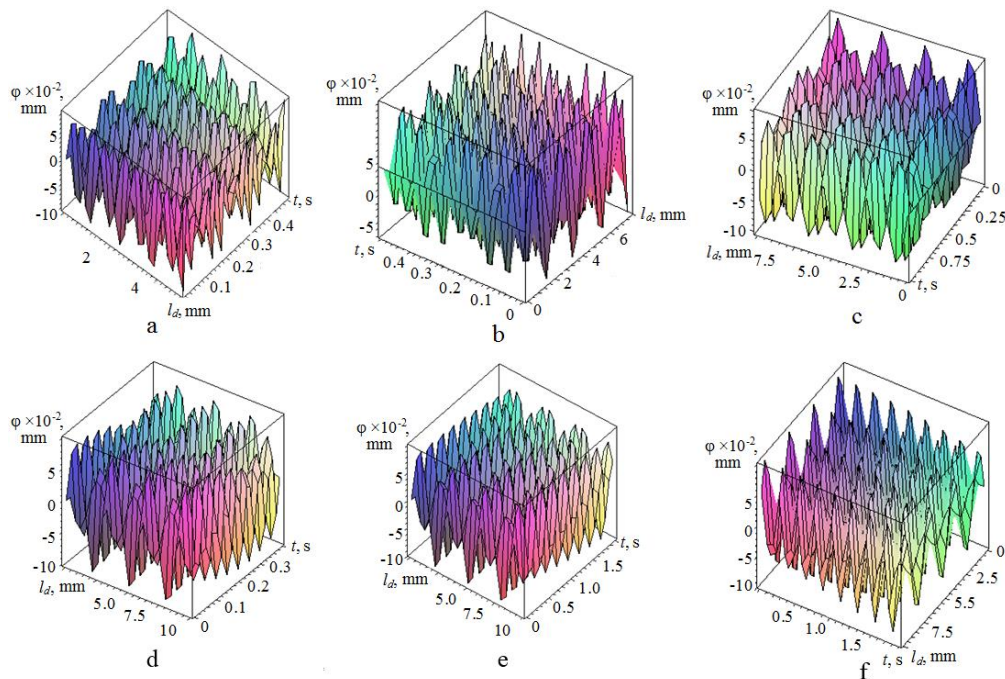
where  $\bar{\Theta}(\tau)$  is

$$\bar{\Theta}(t) = \left\{ S_{0k} \bar{\omega}_k \cos(\bar{\omega}_k t + \varphi_{0k}) + \frac{d}{dt} \left[ \frac{1}{l} \bar{f}_{ph} r_d F_Z \int_0^t \sin\left(\frac{(2k+1)(\bar{x}_0 + V_s\tau)}{2l_d}\right) \sin\left(\frac{(2k+1)\alpha(t-\tau)}{2l_d}\right) d\tau \right] \right\} + \left( \frac{(2k+1)\pi}{2l_d} \right)^4 \frac{\hat{\beta}\pi}{8} \left\{ S_{0k} \sin(\bar{\omega}_k t + \varphi_{0k}) + \frac{1}{l} \bar{f}_{ph} r_d F_Z \int_0^t \sin\left(\frac{(2k+1)(\bar{x}_0 + V_s\tau)}{2l_d}\right) \sin\left(\frac{(2k+1)(t-\tau)}{2l}\right) d\tau \right\} \quad (18)$$

For practical use, the estimation of process dynamics can be performed on the basis of the main mode [20, 21], i.e. on the basis of the ratio

$$\varphi(x,t) = \left\{ \frac{1}{l_d} \bar{f}_{ph} r_d F_Z \left(\frac{3\pi}{2l}\alpha\right)^{-1} \int_0^t \sin\left(\frac{3\pi(\bar{x}_0 + V_s\tau)}{2l_d}\right) \sin\left(\frac{3}{2l_d}\pi\alpha(t-\tau)\right) d\tau + \frac{2l_d\varepsilon}{(2k+1)\pi\alpha} \int_0^t \cos\left(\pi\alpha\frac{3}{2l}(t-\tau)\right) \bar{\Theta}(\tau) d\tau \right\} \quad (19)$$

In Fig. 2 the change in the relative angle of rotation for different characteristics of the process of microrelief is shown.



**Figure 2.** Time variation of the relative torsion angle for different technological parameters of forming microrelief:  
a)  $G = 8 \cdot 10^{10}$  N/m<sup>2</sup>,  $l = 0.5$  m,  $\bar{A} = 0.014$  m<sup>2</sup>,  $F_X = 300$  N,  $V = 0.01$  ms<sup>-1</sup>,  $\bar{f}_{ph} = 0.75$ ,  $r = 0.12$  m;  $\omega = 3$ ;  
b)  $G = 8 \cdot 10^{10}$  N/m<sup>2</sup>,  $l = 0.75$  m,  $\bar{A} = 0.014$  m<sup>2</sup>,  $F_X = 300$  N,  $V = 0.01$  ms<sup>-1</sup>,  $\bar{f}_{ph} = 0.75$ ,  $r = 0.12$  m;  $\omega = 3$ ;  
c)  $G = 8 \cdot 10^{10}$  N/m<sup>2</sup>,  $l = 0.1$  m,  $\bar{A} = 0.014$  m<sup>2</sup>,  $F_X = 300$  N,  $V = 0.01$  ms<sup>-1</sup>,  $\bar{f}_{ph} = 0.75$ ,  $r = 0.12$  m;  $\omega = 3$ ;  
d)  $G = 8 \cdot 10^{10}$  N/m<sup>2</sup>,  $l = 0.1$  m,  $\bar{A} = 0.014$  m<sup>2</sup>,  $F_X = 300$  N,  $V = 0.025$  ms<sup>-1</sup>,  $\bar{f}_{ph} = 0.75$ ,  $r = 0.12$  m;  $\omega = 3$ ;  
e)  $G = 8 \cdot 10^{10}$  N/m<sup>2</sup>,  $l = 0.1$  m,  $\bar{A} = 0.014$  m<sup>2</sup>,  $F_X = 300$  N,  $V = 0.05$  ms<sup>-1</sup>,  $\bar{f}_{ph} = 0.75$ ,  $r = 0.12$  m;  $\omega = 3$ ;  
f)  $G = 8 \cdot 10^{10}$  N/m<sup>2</sup>,  $l = 0.1$  m,  $\bar{A} = 0.014$  m<sup>2</sup>,  $F_X = 300$  N,  $V = 0.05$  ms<sup>-1</sup>,  $\bar{f}_{ph} = 0.75$ ,  $r = 0.12$  m;  $\omega = 6$ .

**Conclusions.** The developed methodology of the analytical description of the technological process of forming microrelief on the inner surface of cylindrical parts and the obtained calculated dependences make it possible to state:

1. The configuration of micro-irregularities depends not only on the angular velocity of rotation of the body and the component of the driving moment that makes the processed body to perform relative, oscillating motion, but also on the elastic vibrations of the body itself.

2. The peculiarity of torsional vibrations is that they are caused by the elastic properties of the body and the external action of force, the point of application of which changes its relative position on the inner cylindrical surface, and therefore the relative displacements of the normal cross-section of the treated body.

3. Amplitude-frequency characteristics of relative torsional vibrations depend on the magnitude of external action and physical and mechanical properties of the treated body. For more rigid body, the frequency of elastic vibrations is higher and the amplitude is slightly lower.

4. The reliability of the obtained calculated dependences is proved by obtaining, in the extreme case, the known ones concerning the process of forming microrelief on a cylindrical surface without taking into account elastic vibrations.

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## **ДИНАМІКА ВІДНОСНИХ КРУТИЛЬНИХ КОЛИВАНЬ ПРИ ФОРМУВАННІ РЕГУЛЯРНОГО МІКРОРЕЛЬЄФУ НА ВНУТРІШНІХ ЦИЛІНДРИЧНИХ ПОВЕРХНЯХ**

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**Резюме.** Наведено результати аналізу сучасних літературних джерел на предмет пошуку математичних моделей, що описують динаміку процесу формування регулярного мікрорельєфу на внутрішніх циліндричних поверхнях деталей, які працюють у важких умовах експлуатації, з метою збільшення їх ресурсу. Встановлено відсутність математичних моделей, що описують даний процес та особливості його здійснення при точковій дії деформуючого елемента на поверхню тіла деталі. Розглянуто рухи інструменту, які супроводжують процес формування регулярного мікрорельєфу на внутрішній циліндричній поверхні тіла деталі та проаналізовано рушійні сили, що супроводжують цей процес. За результатами проведеного аналізу розроблено математичну модель динамічного процесу формування регулярного мікрорельєфу на внутрішній циліндричній поверхні тіла деталі. Особливістю цього процесу є те, що формування мікрорельєфу відбувається зосередженою силою, точка прикладання якої постійно змінюється в радіальному та осьовому напрямках відносно деталі. Відтак, математична модель, яка описує цей процес, буде мати дискретну праву частину. Запропоновано таку дію моделювати за допомогою дельта функцій Дірака з лінійною та часовою змінними, використовуючи метод регуляризації вказаних особливостей, зокрема існуючі методи інтегрування відповідних нелінійних математичних моделей крутильних коливань деталі. Отримано аналітичні співвідношення, які описують ці коливання в процесі формування регулярного мікрорельєфу. Використавши програмне забезпечення Maple, побудовано 3D зміни кута закручування залежно від різних значень вихідних даних. Проведені дослідження дозволяють враховувати крутильні коливання, що особливо актуально для довгомірних циліндричних деталей, таких, як гільзи гідроциліндрів, деталі бурових механізмів та інші.

**Ключові слова:** технологія, циліндрична поверхня, параметри якості, вібраційна обробка, крутильні коливання, математична модель.

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