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MATHEMATICAL MODELING OF HEAT EXCHANGE PROCESSES WHEN HEATING METAL IN A FURNACE

Viktor Aulin; Mykola Bosiy; Volodymyr Kropivnyi; Olexandr Kuzyk; Alena Kropivna

Central Ukrainian National Technical University, Kropyvnytskyi, Ukraine

Summary. *The article presents a mathematical model of the heat transfer process when heating metal in a furnace, which describes the thermophysical processes that occur when using industrial technology for heating castings. A simplified model of convective and radiant heating of metal in a furnace, which is based on conventional differential equations, allows to calculate the heating time of the metal. It allows you to evaluate the process of heating the metal in the furnace and find its optimal parameters. The model is widely used and can be applied to study the process of heating the casting of any metal or alloy in the furnace. The adequacy of the model is confirmed by comparing the obtained results of estimating the parameters of the model with experimental data.*

Key words: *Newton-Richman law, metal heating, variable separation method, Stefan-Boltzmann law, heating furnace.*

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Statement of the problem. Metals and alloys have the wide range of applications in various fields of industry and the extended manufacturing of castings and workpieces of heat-treated metal materials can improve the quality of the final product. Mathematical models of technological processes of metal heating play the important role in production of castings and products which is proved in [1, 2, 4, 5]. Heating furnaces have become widely used in the foundry industry for heating metal [3, 4].

Currently, mathematical modeling of the process of heating metal in furnaces, as the object of study, is an urgent task because it requires more accurate models that take into account all the complex thermophysical processes that occur during heat treatment of castings in furnaces. There are several different models, which together can almost completely describe the process of heating castings [5]. The main ones are the models for estimating the thermal state of the metal (heating model) and the duration of heating, as well as the model for controlling the thermal regime of the furnace. However, in the models, when considering the technological processes of heating metal in the furnace, insufficient attention is paid to the study of heat and mass transfer processes, the analysis of which creates a theoretical basis for improving the design and thermal regimes of thermal furnaces and units, as well as the assessment of ways to improve product quality and reduce fuel consumption and harmful emissions [3, 4, 11].

For research and analysis of these issues, mathematic modelling of heat and mass transfer processes when heating metal in a furnace has been carried out. If the simulation results are confirmed by experimental studies and can be the basis for predicting the processes occurring in the studied objects, the model will be adequate to the object [1, 4].

Analysis of recent research. In most cases, modelling of casted objects is conducted on the basis of the equations of mathematical physics. Features of modeling of technological processes of casting include specific processes of heating of metal in the furnace [1, 2, 4]. The

Corresponding author: Viktor Aulin; e-mail: AulinVV@gmail.com

most convenient is the method based on the physical laws of thermal conductivity: convection according to the Newton-Richman law; radiation according to the Stefan-Boltzmann law; thermal conductivity according to Fourier law [5]. When modelling on the basis of physical laws, the procedure is as follows:

- construction and selection of the structure of the mathematical model and the purpose of modeling;
- estimation of model parameters according to the available data on the observed processes.

Currently, to solve various problems of optimization and control of the technological process of heating metal in the furnace, the mathematical model of heating a massive body, taking into account the effects of heat transfer by radiation and convection is the most widely used [5]. Such models are based on joint solution of external and internal heat transfer problems and heat balance equations under different initial data and boundary conditions. Mathematical models of heating furnaces allow to perform both test calculations of metal heating at a given temperature of the furnace, and determine the heating temperature using computer programs. As a rule, models not more complicated than the one-dimensional equation of thermal conductivity are used to describe the process of metal heating [1, 4, 5]. When using a two- or three-dimensional equation of thermal conductivity there is an urgent need to distribute control over the temperature of the working space of the furnace and all the parameters of heat transfer, which in practice is difficult to implement, and usually not applicable. One working space temperature sensor is usually installed in the furnace zones [1, 2, 4].

In most cases, only zonal thermometers are used to determine the temperature of castings in heat transfer models. The influence of all other parameters of heat transfer can lead to the temperature of the working space in the area of the furnace. In this case, the model of heat transfer can be presented in radiant, radiant-convective, or only in convective forms. Within the physical zone of the furnace, several calculation zones are selected. To take into account the uneven temperature of the working space along the zone, various methods are used to «correct» the readings of zonal thermometers depending on the coordinates along the zone. Clarification of the true value of the total heat transfer coefficient used in the model of heating castings is performed by comparing the measured values of the metal surface temperature in areas with the corresponding calculated values [5].

In [6], a simplified model of the dynamics of metal heating in the system ‘furnace gases – furnace masonry – metal’ without significant deviations from the dynamics of the real object is developed. A mathematical model of optimal control of low-oxidative heating process of metal under the chamber type furnaces conditions is proposed. The criteria of control actions are fuel consumption, specific oxygen consumption and the coefficient of air consumption, which is supplied for afterburning of fuel [7].

In [8], based on the approximation of the instantaneous temperature distribution over the thickness of the heated metal, a mathematical model of heating based on the exponential function is proposed. It can be suggested in the synthesis of the optimal speed control algorithm for the temperature regime of the chamber-type heating furnace.

Analysis of available methods for optimizing the heating process of massive ingots during heat treatment in flame thermal furnaces of chamber type shows a mathematical model [9], which allows to develop an algorithm for calculating the temperature of the heating medium over time without solving differential equations of thermal conductivity. This provides the given temperature distribution in the cross-sections of ingots under heat treatment with two or more levels of constancy. It is shown that the mathematical model and the developed algorithm of calculations of heating thermally massive bodies in flame furnaces of chamber type can be used when considering control of process of heating of metal under heat treatment with three and more levels of constancy [10].

Known problems of mathematical optimization of metal heating for heat treatment in flame furnaces of chamber type are considered [11]. It is found that solving of such type of problems is complicated for the reason of absence of simple model, which would determine the dependence of the final indeces of heating quality on the actions that manage in the system ‘heating gases – masonry – metal’. With some assumptions, the simulation results can be used only for qualitative analysis of the thermal performance of furnaces.

Since the adequacy of models in the real process is determined by identification methods, in this case, information about the temperature of the metal is a source of assessing the quality of the model and the development of simple and reliable identification algorithms. To predict the heating time of castings and workpieces, the estimation of the mathematical model of the time interval between successive feeding of two castings of one batch of metal into the furnace is used. Thus, the construction of a mathematical model of the heat transfer process in heating furnaces is certainly an urgent task.

Statement of the problem. The purpose of this paper is to build a refined mathematical model that takes into account all the processes of heat transfer when heating metal in the furnace.

Problem solution. Knowledge of heat and mass transfer processes in modern technology of heat treatment of materials plays an important role, as the development of heat transfer is one of the main sections of modern basic science. It should be noted that the methods of studying the processes of mass and energy transfer are constantly evolving and allow to clarify the essence of thermal technologies [12]. We analyze the works on the basis of which the refined mathematical model of heat exchange processes is built.

The analysis of mathematical modeling methods for thermal work of heating and thermal furnaces showed that currently, along with traditional methods of solving thermal conductivity problems, software packages are widespread to take into account a number of complex thermophysical and hydrodynamic processes occurring when heating metal in furnaces [13].

In [14], the structural schemes of adaptive control systems of heating furnaces with the detailed analysis of objects, and also mathematical models of heat and mass transfer processes for use in these systems are resulted. The method of assembling models is considered, as well as variants of their application for optimization of metal heating modes.

In [15], a mathematical model of metal heating and the choice of furnace temperature, which minimizes the heating time, was developed, and the issue of endurance during heat treatment of alloy steel was considered. The simulation results show an increase in the quality of metal heat treatment, reduction of heat treatment time and decrease of total fuel consumption [16].

It is found that mathematical models are built mostly for one-factor systems (i.e. for systems with one independent variable) and not often for systems with more independent variables.

First, for a mathematical model of the type y=f(x) make the differential equation of this dependence. Then use the relevant fundamental laws of physics, chemistry, physical chemistry and other natural sciences are used, the situation is considered at some arbitrary point of this dependence. Giving the argument an infinitesimal increment dx, write the corresponding infinitesimal increment of the function dy and obtain a differential equation, the integration of which, taking into account the specific conditions of the problem gives a theoretical mathematical model.

The problem of constructing a mathematical model for multifactor systems is much more complex. An example of such a problem is the construction of a theoretical mathematical model of nonstationary thermal conductivity for simple-shaped bodies (plate, cylinder, sphere). Mathematical models on this issue are widely used in heat engineering of casting technological processes [2].

Consider the mathematical model of heating the metal in the furnace for the condition that the heated body is ‘thin’, i.e. its thermal conductivity ratio λ is very high and the heat energy from the furnace to the metal is transferred according to the Newton-Richman’s law at the constant temperature of the furnace (Tf = const).

At the moment τ from the start of heating during the time $d\tau$, from the furnace to the metal, according to the presented scheme (Fig.1), great amount of heat is transferred, which is equal:

$$\delta Q = \alpha\left(T_{\mathrm{f}} - T_{\mathrm{m}}\right)F d\tau\,, \tag{1}$$

where α is the effective effective heat transfer coefficient by convection from furnace gases to metal, W/m^2·K;

T_{f} and T_{m} are the temperatures of furnace and the metal, K;

F is heat-absorbing metal surface, m^2.

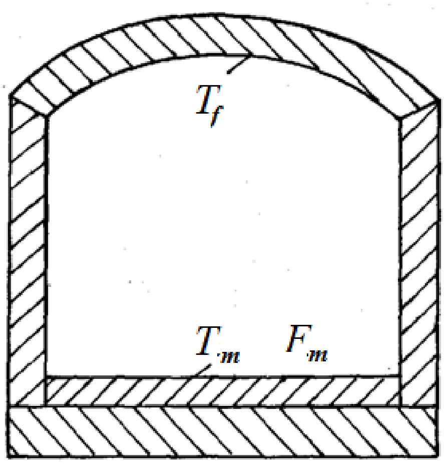

Figure 1. Scheme of heating the metal in the furnace

The amount of heat δQ, which heats the metal to a temperature of dT_{m}, K is:

$$\delta Q = mcdT_{\mathrm{m}}\,, \tag{2}$$

where m and c are the mass and heat capacity of the metal.

We equate the right-hand sides of equations (1) and (2) and obtain the differential equation of the process, which connects two variables – heating time τ and metal temperature T_{m}:

$$\alpha\left(T_{\mathrm{f}} - T_{\mathrm{m}}\right)F d\tau = mcdT_{\mathrm{m}}\,. \tag{3}$$

Equation (3) is solved by the method of separating variables:

$$\int_{0}^{\tau} d\tau = \left(mc/\alpha F\right)\int_{T_{\mathrm{m.initial}}}^{T_{\mathrm{m.fin}}} dT_{\mathrm{m}}/\left(T_{\mathrm{f}} - T_{\mathrm{m}}\right). \tag{4}$$

where $T_{\mathrm{m.fin}}$ and $T_{\mathrm{m.initial}}$ are the final metal temperature and the initial metal temperature, K.

Integrate equation (4) and obtain the expression to determine the duration of heating:

$$\tau = \left(mc/\alpha F\right)\ln\left[\left(T_{\mathrm{f}} - T_{\mathrm{m.initial}}\right)/\left(T_{\mathrm{f}} - T_{\mathrm{m.fin}}\right)\right]. \tag{5}$$

Expression (5) will be a theoretical model of metal heating in a furnace.

The mass of metal is represented as the product of its density by volume $\left(m = \rho V\right)$, and the ratio of volume to surface area F – as the reduced thickness of the body $S_{\mathrm{rt}} = V/F$. Given this, equation (5) is:

$$\tau = \left(\rho S_{\mathrm{rt}} c/\alpha\right)\ln\left[\left(T_{\mathrm{f}} - T_{\mathrm{m.initial}}\right)/\left(T_{\mathrm{f}} - T_{\mathrm{m.fin}}\right)\right]. \tag{6}$$

The value of the effective coefficient of heat transfer by convection α is known and can be determined from the literature or by the formula [2]

$$\alpha = C_{\mathrm{r}} 10^{-8}\left(T_{\mathrm{f}} + T_{\mathrm{m}}\right)\left(T_{\mathrm{f}}^2 + T_{\mathrm{m}}^2\right), \tag{7}$$

where C_r is a reduced radiation coefficient in the system ‘furnace-metal’, calculated by the formula [2]:

$$C_{\mathrm{r}} = 1/\left[\left(1/C_{\mathrm{rad}} - 1/C_{\mathrm{o}}\right)\varphi_{\mathrm{rad.sm}} + 1/C_{\mathrm{o}} + \left(1/C_{\mathrm{sm}} - 1/C_{\mathrm{o}}\right)\varphi_{\mathrm{sm.rad}}\right], \tag{8}$$

where C_{rad}, C_{sm} and C_o – coefficients of radiation of the furnace inner surface of the masonry of the furnace working space, the surface of the metal and absolutely black body;

$\varphi_{rad.sm}$ and $\varphi_{sm.rad}$ are corresponding angular coefficients.

If the metal in the furnace is heated in accordance with the law of Stefan-Boltzmann, the elementary amount of heat of the process is equal to:

$$\delta Q = C_{\mathrm{r}}\left[\left(T_{\mathrm{f}}/100\right)^4 - \left(T_{\mathrm{m}}/100\right)^4\right]F_{\mathrm{m}} d\tau . \tag{9}$$

Equate the right parts of equations (9) and (2); after integration by the method of separated variables, we obtain:

$$\int_0^{\tau} d\tau = \left(mc/C_{\mathrm{r}} F_{\mathrm{m}}\right)\int_{T_{\mathrm{m.initial}}}^{T_{\mathrm{m.fin}}} dT_{\mathrm{m}} \Big/ \left[\left(T_{\mathrm{f}}/100\right)^4 - \left(T_{\mathrm{f}}/100\right)^4\right]. \tag{10}$$

After the corresponding integration of equation (10), the mathematical model (5) of the duration of heating the metal in the furnace takes the form:

$$\begin{gathered}\tau = \left(\rho S_{\mathrm{rt}} c 10^8/C_{\mathrm{r}} T_{\mathrm{f}}^3\right)\left\{0{,}5\left[\mathrm{arctg}\left(T_{\mathrm{m.fin}}/T_{\mathrm{f}}\right) - \mathrm{arctg}\left(T_{\mathrm{m.initial}}/T_{\mathrm{f}}\right)\right] + \right. \\ \left. + 0{,}25\ln\left[\left(1 + T_{\mathrm{m.fin}}/T_{\mathrm{f}}\right)\left(1 - T_{\mathrm{m.initial}}/T_{\mathrm{f}}\right)/\left(1 - T_{\mathrm{m.fin}}/T_{\mathrm{f}}\right)\left(1 + T_{\mathrm{m.initial}}/T_{\mathrm{f}}\right)\right]\right\}.\end{gathered} \tag{11}$$

Given these assumptions, models (5) and (11) can be attributed to models that describe the heating of ‘thin’ bodies. If the body is ‘massive’, then the mathematical model, in addition to equations (5) or (11), takes into account the relations given in [2]:

$$\tau_{\text{ф}} = m\tau ; \tag{12}$$

$$m = 1 + 0{,}5\mathrm{Bi}\ , \tag{13}$$

where $\tau_ф$ is the actual heating time of the 'massive' body;
τ is the time determined by formulas (5) or (11);
$\mathrm{Bi} = \alpha S_{\mathrm{rt}}/\lambda$ – Bio-number;

According to the obtained mathematical model, the time of heating the casting in the heating well of the furnace was estimated: the size of the casting is 0.3x0.3x0.6 m; the size of the vertically placed furnace well, which is heated is 0.6x0.6x1m at a constant furnace temperature T_f = 1873 K (1600°C), the final metal temperature $T_{m.fin}$=1453K (1180°C), the initial metal temperature $T_{m.initial}$ = 291 K (18°C)), metal density ρ = 7200 kg/m^3, heat capacity of the metal c = 0.7 kJ/kgK, the coefficients of radiation of the metal and masonry, respectively, C_1=4.0 and C_2=4.7 W/m^{2}K4, the thermal conductivity of the metal λ=20 W/mK. The heating time of the casting according to formula (11) is τ= 0.28 hours, the effective heat transfer coefficient α=714 W/m^{2}K, the number Bi=2.35, the parameter m=2.17. The actual heating time is τ=0.62 h. The calculated values of the metal heating time τ_e = 0.61 h obtained in the given example are close to the real $\tau \approx \tau_e$, i.e. the model can be considered adequate.

Conclusions. A model of convective and radiant heating of metal in a furnace is constructed on ordinary differential equations which allows to calculate the heating time of castings from different alloys. It is shown that the model of heating metal in the furnace taking into account the constant temperature of the furnace can be used to solve problems of improving the efficiency of the furnace equipment complex, as well as to determine the heat potential of furnaces exhaust. Estimation of the results accuracy was performed by comparing the calculated values of the castings heating time in the furnace with the available experimental data, which justifies the reliability of the obtained results with sufficient accuracy.

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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ПРОЦЕСІВ ТЕПЛООБМІНУ ПРИ НАГРІВАННІ МЕТАЛУ В ПЕЧІ

Віктор Аулін; Микола Босий; Володимир Кропівний; Олександр Кузик; Альона Кропівна

Центральноукраїнський національний технічний університет, Кропивницький, Україна

Резюме. *Математичне моделювання процесу нагрівання металу в печах, як об'єкт дослідження, є актуальним завданням за потреби більш точних моделей, беручи до уваги складні теплофізичні процеси, які протікають при тепловій обробці виливків у печах. Відомі натепер моделі можуть описати процес нагрівання виливків у печах за допомогою рівнянь теплопровідності. Основні з них – це моделі оцінювання теплового стану металу (модель нагрівання) й тривалості нагрівання, а також модель управління тепловим режимом печі. Однак у цих моделях, при розгляді технологічних процесів нагрівання металу в печі, недостатньо приділено уваги вивченню тепломасообмінних процесів, аналіз яких створює теоретичну базу для вдосконалення конструкцій і теплових режимів термічних печей і агрегатів, можливостей доцільного впливу на структуру і властивості металів і сплавів, а також для оцінювання шляхів підвищення якості продукції та зниження витрат палива і шкідливих викидів. Виходячи з цього, представлено математичну модель процесу теплообміну при нагріванні металу в печі, яка описує теплофізичні процеси, що мають місце при використанні промислової технології нагрівання виливків. Спрощена модель конвективного і променевого нагрівання металу в печі, яка побудована на звичайних диференціальних рівняннях, дозволяє розрахувати час нагрівання металу. Вона дає змогу оцінити процес нагрівання металу в печі та знайти його оптимальні параметри. Модель передбачає широке застосування й може бути використана для вивчення процесу нагрівання виливка з будь-якого металу чи сплаву в печі. Адекватність моделі підтверджується порівнянням отриманих результатів оцінювання параметрів за моделлю з експериментальними даними. Показано, що модель нагрівання металу в печі з урахуванням сталої температури печі може бути використана для вирішення завдань підвищення ефективності роботи комплексу обладнання печі, а також для визначення потенціалу використання теплоти відхідних газів нагрівальних печей.*

Ключові слова: *закон Ньютона-Ріхмана, нагрівання металу, метод відокремлення змінних, закон Стефана-Больцмана, нагрівальна піч.*

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