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Modal analysis of aircraft engine blisks

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Abstract: *The article deals with the actual problem of ensuring the vibrational reliability of aircraft engine blisks by conducting modal analysis, which makes it possible to determine natural vibration frequencies and modes. The localization of vibrations, increased resonance stresses and threat of disk destruction have been subjected to analysis due to heterogeneous geometric configuration of blades caused by machining errors and operational wear. The paper presents a comparative analysis of experimental natural vibration frequencies and mathematical modeling by the finite element method (FEM) using a specialized software demonstrating high convergence of results. Particular attention is paid to evaluating the effectiveness of various technological refinements for both compressor centrifugal impellers and axial stages. These modifications include trimming the leading edge and tailoring the profile thickness into several distinct shapes. The primary objective of these adjustments is to shift the natural vibration frequencies beyond the engine's resonance operational range. The study covers the analysis of the influence of centrifugal forces, which increase the dynamic stiffness of the structure, aerodynamic loads, as well as the temperature factor, which significantly reduces frequencies due to changes in the material's elasticity moduli under high-temperature operation. To evaluate resonance conditions, Campbell diagrams are used, which make it possible to graphically determine the intersection points of natural frequencies with excitation harmonics from the engine's structural elements. The practical part is based on the analysis of the dynamic characteristics of the AI-450MS, D-36, and PZL-10W engine components to confirm the possibility of avoiding resonance zones through structural modifications. Additionally, the role of harmonic analysis is considered, which, unlike modal analysis, makes it possible to take into account the system damping and determine the amplitude-frequency characteristics of the blade under influence of specific exciting forces.*

Keywords: *natural vibration frequency, blisk, centrifugal impeller, modal analysis, finite element method, Campbell diagram.*

1. INTRODUCTION

Modern aircraft engine manufacturing is focused on producing the units with higher service life and economic efficiency. One of the most critical components of an aircraft engine is the compressor rotor, which operates under extreme rotation frequency (20,000 rpm and more) and aerodynamic loads. To ensure reliability in such conditions, the increasing use is made of bladed disks (blisks), where blades and disk form a single whole.

The use of blisks makes it possible to significantly reduce the rotor weight and eliminate problems related to the blade root attachment. Such products are most often made of two-phase titanium alloys (for example, VT3-1, VT6, VT8 (VT8-1), VT9), which have high specific strength and corrosion resistance [1].

A transition to one-piece machined structures results in decreasing the system damping capacity. Due to no frictional interaction in the contact areas between the blade roots and the disk, the vibration energy is virtually not dissipated, which increases the blisk sensitivity to resonance loads [2].

The natural vibration frequency (NVF) is one of the parameters to be inspected in the course of manufacturing both compressor centrifugal impellers (CCI) and axial stages of gas turbine engines. Shape errors and deviations from dimensions caused by machining, including finishing, even those that are within the tolerance range, operating wear (such a phenomenon can also be observed during operation), lead to heterogeneous geometric configuration of blades within one part. This in turn could lead to increased resonance stresses acting on individual blades during vibrations in the course of the engine operation [3].

Despite the minimum differences between NVFs of the blades, the forced response amplitude significantly exceeds the values of ideally tuned systems [3, 4]. Due to the localization of vibration energy [5, 6], there is a risk of premature multi-cycle fatigue of the blades and initiation of cracks in the disk root attachments, which threatens the integrity of the entire structure [7]. Leading Ukrainian specialists were engaged in the study of the phenomenon of natural frequency mistuning of compressor blades, its prediction and development of frequency tuning methods: A. P. Zinkovsky, O. L. Derkach, I. H. Tokar, I. F. Kravchenko, V. M. Merkulov, K. V. Savchenko, O. O. Larin and others. A significant contribution to the study of this problem was also made by foreign authors: C. Pierre, D. J. Ewins, M. P. Costanier, B. I. Epureanu and M. Imregun.

If higher dynamic stability of the compressor axial stages with inserted blades is partially achieved by distributing them by weight moment, i.e. by weighing and installing them in the disk in the appropriate sequence to ensure the minimum output imbalance during balancing, such a selection is impossible for blisk-type parts. Therefore, there is a task to ensure high geometric uniformity of blisk blades.

The NVF value makes it possible to evaluate the accuracy of manufacturing and, if necessary, make a decision on the refinement of individual blades. The permissible NVF range for blades is determined by collecting statistics on manufactured parts and comparing the obtained NVF values with the design ones. To ensure that the natural vibration frequency of the compressor centrifugal impellers and axial stages avoids resonance frequencies that can cause increased dynamic stresses at the engine operating conditions, A.P. Zinkovsky recommends [4] to maintain the NVF beyond the 10% interval of the vibration excitation frequency at the operating conditions.

The aim of this study is to develop and evaluate a comprehensive methodology for ensuring the vibration reliability of aircraft engine blisks and centrifugal impellers through geometric modifications of blade airfoils, providing controlled shifts in natural frequencies to mitigate resonance risks.

To achieve this aim, the following objectives were formulated:

- To construct advanced finite element models for the modal analysis of integrally bladed rotors and validate their accuracy against experimental dynamic data.
- To investigate the regularities of the influence of alternative geometric profiling strategies and blade edge trimming configurations on the redistribution of natural vibration frequencies across the primary vibration modes.
- To assess the joint sensitivity of the structural dynamic characteristics to severe operational factors, including centrifugal stiffening, aerodynamic loading, and thermal degradation of the material's elastic properties.
- To perform a comprehensive frequency-matching analysis using Campbell diagrams to identify potential resonance intersections and establish safe operational speed ranges for high-speed turbomachinery components.

2. METHODS FOR DETERMINING NATURAL VIBRATION FREQUENCY

The natural vibration frequency of the centrifugal and axial blisks is most often determined by the first mode of natural vibrations. One of the multi-purpose tools used for automatic frequency inspection of various blisk blades is the «Mikat-ChKM» unit (Fig. 1) [8].

Figure 1. «MIKAT-ChKM» unit for automatic frequency inspection.

In addition to experimental methods for determining the NVF, mathematical modeling by the finite element method (FEM) using specialized software is widely used and it can determine the NVF with high correspondence to experimental results [9, 10].

Modal analysis for determining the natural vibration frequencies and modes is performed on the basis of the equation of free vibrations [11, 12]:

$$[M]\{\ddot{u}\} + [\mathrm{C}]\{\dot{u}\} + [K]\{u\} = \{0\}, \quad (1)$$

where [M], [C] and [K] are matrices of mass, damping and stiffness, respectively, {u} is the vector of nodal displacements, $\{\dot{u}\}$ is the velocity vector and $\{\ddot{u}\}$ is the acceleration vector of body points.

A distinctive feature of modal analysis is that external forces and damping of the system are assumed to be zero, so the equation of free vibrations in matrix form is as follows:

$$[M]\{\ddot{u}\} + [K]\{u\} = \{0\}, \quad (2)$$

The stiffness and mass matrices are determined by the following equations:

$$[K] = \sum_{i=1}^{ne}[K_i^e]; \quad (3)$$

$$[M] = \sum_{i=1}^{ne}[M_i^e], \quad (4)$$

where ne is the number of system components, $[K_i^e]$ and $[M_i^e]$ are matrices of masses and stiffness of individual elements, respectively.

Free vibrations for the linear system are determined as:

$$\{\mathrm{u}\} = \{\varphi\}_\mathrm{i} \cos \omega_\mathrm{i} \mathrm{t}, \quad (5)$$

from which:

$$\{\dot{u}\} = -\omega_i\{\varphi\}_i \sin \omega_i t; \quad (6)$$

$$\{\ddot{u}\} = \omega_i^2\{\varphi\}_i \cos \omega_i t, \quad (7)$$

where $\{\varphi\}_i$ is the i-th eigen vector representing the vibration mode at the i-th eigen frequency, ω_i is the i-th eigen circular frequency, t is time.

Substituting equations (5–7) into (1), we obtain [9]:

$$\left(-\omega_i^2[M] + [K]\right)\{\varphi\}_i = \{0\}. \quad (8)$$

This equation has two solutions:

$$\begin{gathered}\{\varphi\}_i = 0 \text{ i} \\ \left(-\omega_i^2[M] + [K]\right)=0.\end{gathered} \quad (9)$$

Assuming that n is the matrix order, the equation is solved by finding the nth-order polynomial with n roots: $\omega_1^2, \omega_2^2, \ldots, \omega_n^2$. These roots are called the eigen values of the equation. The circular eigen frequency of vibrations ω_i is found by extracting the root from the eigen values. The values of the circular eigen frequencies of vibrations ω_i and eigen frequencies of vibrations fi are related to each other through the following equation:

$$f_i = \frac{\omega_i}{2\pi}. \quad (10)$$

By substituting the natural vibration frequencies into equation (8), the corresponding eigen vectors $\{\varphi 0\}, \{\varphi 1\}, \ldots, \{\varphi n\}$ describing the natural vibration modes are found.

The combination of analytical methods with modern finite element modeling algorithms ensures a high-accuracy prediction of the blisk vibration state. This minimizes the volume of expensive experimental studies and provides the possibility of targeted control of the frequency spectrum of blades.

3. ANALYSIS OF NATURAL VIBRATION FREQUENCY OF ROTOR PARTS OF AIRCRAFT ENGINES

To ensure the refinement of compressor centrifugal impeller blades, the researchers from State Enterprise «Ivchenko-Progress» (Zaporizhzhia, Ukraine) evaluated three options: trimming the blade angles at the leading edge from the root section on the blade periphery side; reducing the blade thickness from the suction surface side on the blade periphery; and a combination of these two measures. Combining both refinement options significantly increases the blade NVF by up to 15% of its initial value. The relationship between the amount of material removed and the frequency shift exhibits a linear character that remains consistent across different blades. However, trimming the leading edge alone proves less effective than modifying the blade profile thickness on the suction surface side.

The same patterns are indicated in the study [8], where the influence of refinement of the AI 450-MS engine compressor centrifugal impeller made of titanium alloy VT8-1 was researched, and it is noted that the metal removal from the leading-edge side increases the blade NVF, but from the trailing edge side – decreases the blade NVF. It is worth noting the high value of the NVF for the first mode of blade vibrations, which was in the range from 2775 to 2905 Hz.

Changing the shape of CCI and rotor blades of axial compressors at the design stage should take into account the impact not only on the first bending mode, but also on other modes of natural vibrations, since they can fall into the resonance areas of the engine operating conditions [8,12]. Moreover, the blade NVF can be affected by long-term operation of products, as a result of which, the action of abrasive and erosive wear, for example, or permissible

reworks of damaged blades during overhaul (in particular, chamfers on the leading edges, recesses at the leading/trailing edges), can change the blade geometry, quality of surfaces and, accordingly, affect the NVF, damping properties, and safety margin. First of all, this is due to thin leading and trailing edges of the rotor blade, which take the load caused by the incoming air. Damage to working surfaces (nicks, scratches, etc.) caused by such interaction leads to occurrence of stress concentrators, which increase the probability of their destruction and deformation due to vibration loads.

All this requires the NVF to be shifted within the safe frequency range, which leads to increasing the costs of manufacturing (refinement) of blades and, as a result, to a fairly large percentage of parts with irreparable discrepancies (30–40%) [12].

The study [12] researches the effectiveness of the influence of various options used for technological refinement of the blade lateral surfaces within the tolerance limits for dimensions Cmax, C1, C2 and B, on the NVF values of the D-36 engine low-pressure compressor (LPC) stage 1 rotor blades made of titanium alloy VT3-1. The results of determining the NVFs of blades with changed geometry are given in Table 1.

Table 1. Design values of NVF for blades with changed geometry.

Shape of blade airfoil refinement	Natural vibration frequencies, Hz		
	f1	f2	f3
Straight wedge	366.40	1129.74	1590.47
Inverse wedge	338.69	1112.11	1515.98
Equidistant shape	352.30	1122.13	1554.83
X-shape	339.47	1088.69	1506.22
Rhombus shape	361.88	1153.05	1598.82

The arithmetic mean values of the previously measured NVFs for the first three modes of blade vibrations were as follows:

- f1=335.69 Hz – frequency of the first bending mode of natural vibrations;
- f2=1113.35 Hz – frequency of the first torsional mode of natural vibrations;
- f3=1502.69 Hz – frequency of the second bending mode of natural vibrations.

According to Table 1, for f1 the largest change in the NVF occurs when using the "straight wedge" option and is 30.71 Hz, or 8.3% of the initial value. For f2 and f3, the most effective option is "rhombus-shaped", in which the NVF change was 39.7 Hz, or 3.5% of the initial value for second mode of natural vibrations and 96.13 Hz, or 6% of the initial value for the third mode.

In most cases, the NVF value increases during refinement due to decreased blade stiffness, but in some cases, such as the "X-shaped" option for changing thickness for the first torsional mode of natural vibrations, it may even decrease.

To assess the possibility of obtaining the comparable NVF values when using mathematical modeling by FEM, the author developed geometric models and performed modal analysis using the Altair SimSolid software package [12] for all options of the blade airfoil shape [12]. According to the modeling results, the difference was determined between experimentally obtained values of the blade NVF and the FEM modal analysis data, and the above difference was a maximum of 1.69% for the fifth option of model refinement for the first mode.

The author [12] studied the influence of blade length L on the NVF value. Two options for changing the blade length relative to the initial value are proposed: 1/9L (10% of the length) and 2/9L (20% of the length). It is expected that a decrease in the blade length will lead to an increase in its stiffness, and accordingly, the blade NVF will increase. The NVF versus its height graph for three shapes is shown in Fig. 2a, from which it can be seen that the largest change in the NVF will be for the first shape by 59% and for the third shape by 58%.

However, standard compressor rotor blade overhaul technologies do not permit such significant height modifications. To ensure a uniform tip clearance between the blades and the engine stator, the assembled compressor rotor stages undergo radial grinding. This procedure adheres to the overhaul documentation, where the final rotor diameter can be reduced across several repair categories, typically resulting in a minor variance of 0.2–0.3 mm. Nevertheless, the above function can be used for designing or upgrading new engines, although such significant blade height modification will accordingly cause the required adjustment of the adjacent stages.

The study of the influence of chord reduction by 10 and 20% on the leading edge side for 1/3 of the blade height on its NVF showed insignificant interrelation, and the graph with modeling results is given in Fig. 2b and Fig. 2c. The maximum change is for the first (8%) and second modes of the NVF (7%).

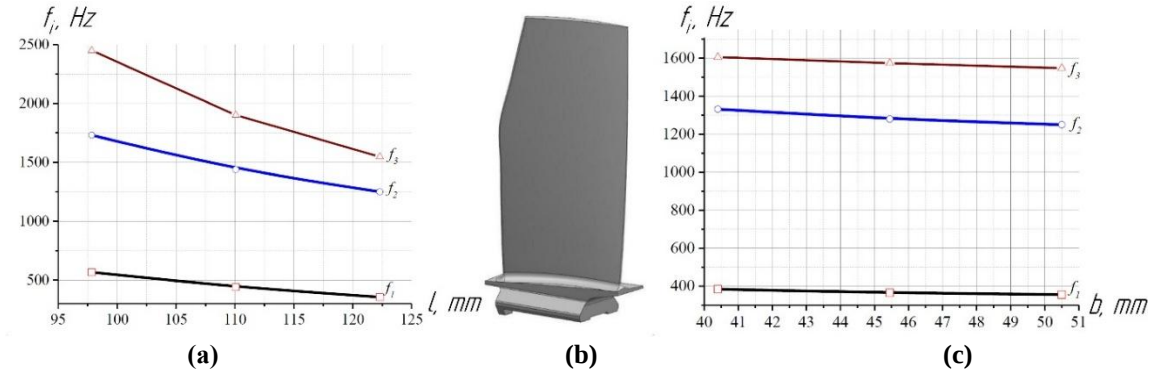

Figure 2. Blade NVF versus its height for three modes – (a); blade model with profile chord reduced by 20% in the 1-st section – (b); blade NVF versus airfoil chord for the first three modes in the 1-st section – (c).

The author [12] suggests determining the gas turbine engine rotor blade resonance conditions graphically using the Campbell diagram, plotting and use of which is also described in studies [12 - 16]. The abscissa axis shows the rotor speed, the range of engine operating conditions is plotted from Idle (nidle) to Maximum (nmax) power rating.

The rotor blade NVF is plotted along the ordinate axis. When the compressor rotates, the blade is affected by centrifugal forces, which are equivalent to increased stiffness of the blade and, accordingly, they will increase its NVF value. The higher the rotational frequency, the higher the difference between static fst and dynamic fdyn values of the blade NVF, the value of the latter is determined by the following formula [12]:

$$f_{dyn} = \sqrt{f_{st}^2 + B \cdot n_c^2}, \tag{11}$$

where $B = 1 + \frac{R}{L}$ is the factor that takes into account centrifugal forces;
R is radius of the disk on which the rotor blade is cantilevered;
L is blade height;
nc is rotational speed of the rotor, 1/s.

The frequency values of excitation harmonics fk, which are determined by multiplying the harmonic number k by the engine rotational frequency, are also plotted:

$$f_k = k \cdot n_c. \tag{12}$$

The author [12] notes that the most dangerous sources of vibration excitation are those located upstream of the impeller. For the D-36 engine, such elements are 6 struts of the LPC front bearing support housing (k=1; 2; 3; 6), 57 straightening vanes of the LPC (k=1; 57), and 35 rotor blades of LPC stage 1 (k=1; 5; 7; 35).

The resonance frequency is the frequency at which the lines of NVF for each vibration mode and those of excitation frequency intersect. As shown in Fig. 3, none of the resonance frequency points (highlighted by red dots) falls within the operating range of the compressor rotor rotational speed, but for the first vibration mode and the 3rd harmonic, as well as for the second vibration mode and the 7th harmonic, excitation occurs almost on the border with the operating range.

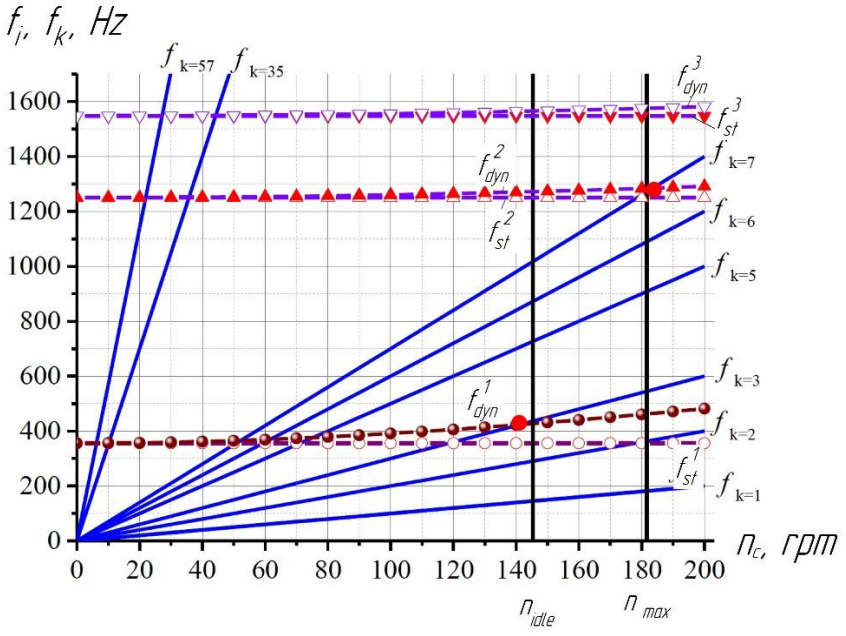

Figure 3. Campbell diagram for LPT stage 1 rotor blade of D-36 engine [12].

The authors [14] performed a modal analysis of the blisk taking into account the data previously determined using the ANSYS CFX computational fluid dynamics software regarding the pressure distribution over the blades during operation under centrifugal force caused by the blisk rotation at rotational speed of 18,000 rpm.

The study [14] indicates that the blisk modal characteristics depend on the thickness and, accordingly, the stiffness of the disk. It is determined that changing the disk thickness can lead not only to a change in the blisk natural vibration frequency value, but also to a change in the sequence of natural vibration modes. The conducted modal analysis of the blisk showed that the general modes are formed from a combination of vibration modes of the disk and blades. The disk performs compatible vibrations with blades for the first six NVFs, and bending vibrations of the blades according to the first mode with different phase angles dominate for the next 22 NVFs, which are reflected in the proximity of the NVF values. According to the results of plotting the Campbell diagram, the resonance rotational speed (13,853 rpm) was determined, which is closest to the operating rotational speed (14,500 rpm) and corresponds to the fourth NVF of the blisk.

Based on the figures showing the natural vibration modes in the study [14], the authors set the displacement limits by the hub inner diameter, although most often such stages are either welded together into drums using electron beam welding, or tightened by bolted joints, which are located above the average diameter of the disk and, accordingly, it is in these very places that zero displacements shall be set.

The issues of the significance of the influence of aerodynamic loads and rotor rotational speed on the natural vibration frequency of the PZL-10W turboshaft engine axial compressor stage 1 blade made of alloy EI961 were considered in the study [16]. The authors determined that the aerodynamic flow builds up the pressure acting on the blade surface and causes its deflection and appearance of stresses of up to 984 MPa at the maximum rotor rotation frequency of 31,486 rpm, but almost does not affect the blade NVF. The first bending NVF obtained by

the FEM using the ANSYS v.19 software was 789.62 Hz. The second and third NVFs are of bending-torsional vibration mode and equal 2521.6 Hz and 3640 Hz, respectively.

As shown above, centrifugal forces increase the blade natural vibration frequency due to its increased stiffness, while the influence of aerodynamic loads decreases with increasing compressor rotational speed, which is observed for all three NVFs. The NVF value under centrifugal forces for the first mode without applying aerodynamic load is 789.62 Hz at 0 rpm, 841.25 Hz at 11,000 rpm, 985.30 Hz at 22,490 rpm, and 1136.9 Hz at 31,486 rpm. Taking into account the aerodynamic load, these NVFs are 789.62 Hz; 841.45 Hz, 985.43 Hz and 1137 Hz, respectively, for each rotational speed.

While the compressor blisks under study are fabricated from titanium alloys operating within lower thermal ranges, a turbine blade investigation from the literature [18] is considered as an extended comparative benchmarking example. This allows for a clearer demonstration of how a profound shift in the material's elasticity parameters alters the structural frequency spectrum under high temperatures.

The study [18] researches the influence of operating temperature on the NVF of turbine blades made of a typical heat-resistant nickel alloy. Two cases of blade operation were modeled: at temperatures of 20 and 1100˚C. A significant decrease in the blade NVF is noted with increasing temperature (ref. Table 2), which is caused by decreased shear modulus G and elasticity modulus E with increasing temperature.

Table 2. Blade NVFs for various temperatures.

Design NVF values		
Vibration mode No.	At t=20˚C F20˚C, Hz	At t=1100˚C F1100˚C, Hz
1	11252	9177.9
2	18946	15571
3	24130	19706
4	32011	26247

Wang Di [16] determined the NVF of the compressor centrifugal impellers using the finite element method. The finite element model consists of 981522 elements and 192280 nodes. The NVF was determined taking into account the impeller rotational speed and the corresponding influence of centrifugal forces. Modal analysis was performed by the Block Lanczos method and shows a sufficiently high convergence between the results of mathematical modeling and those of experimental research, the maximum difference between their NVFs was 8.6% for the first mode. The calculated value of the NVF of the compressor centrifugal impeller blade for the first bending mode of the blade was 1792.1 Hz, while the experimentally obtained NVF of the compressor centrifugal impeller blades was 1637 Hz.

The NVF of any element is the frequency, to which the value of the largest amplitude of vibrations of this element corresponds for the appropriate mode. The vibration excitation occurs starting from a certain frequency preceding the element NVF, it is clearly traced by a gradual increase in the vibration amplitude and, after reaching the peak, its decrease. The vibration amplitude values at any frequency are obtained from the results of harmonic analysis, where the appropriate exciting force value is set in a certain direction.

Based on the above, it can be concluded that the vibration excitation occurs in a certain frequency range, and the vibration amplitude value changes within this range. Taking into account the experience of research of the natural vibration frequency of various parts and units, it can be concluded that both the frequency range limits, in which the vibration excitation occurs, and the vibration amplitudes can differ significantly from each other. Thus, the vibration

excitation frequency range limits can be wider than those for changing the NVF of blades during their refinement, so such actions may be ineffective.

A harmonic analysis makes it possible to obtain the blade's response value to the influence of a harmonically acting force and, as a result, an amplitude-frequency characteristic (AFC), that is actually a graph of displacement of selected points of the blade along a certain axis in a given frequency range. Unlike modal analysis, harmonic analysis takes into account the system (part) damping, and the acting force or some other types of loads (moments, stresses, etc.) are also specified.

Neighboring natural frequencies of the blisks are characterized by unequal intensity of vibration excitation. In particular, the amplitude response in the first and second natural modes can differ by an order of magnitude. This indicates that even if resonance occurs at certain frequencies, the amplitude level and, as a result, the corresponding dynamic stresses may not reach critical values for the structural strength.

Taking into account that the CCI hub and axial stage disk have stiffness much higher than stiffness of thin-walled blades, the vibration amplitudes of the above elements may have lower values. However, if the disk or hub NVFs are close to the operating frequencies of the blades, the "coupling" of modes will occur. A harmonic analysis makes it possible to estimate the vibration amplitude values of the elements at different NVFs and make a decision on the expediency of simplifying the model to analyze the blade only. In this case, zero displacements are set at the blade-to-hub mating point, which accelerates calculations significantly. However, a preliminary determination of the hub NVF value is still necessary.

It is critical to note, however, that while geometric modifications such as leading-edge trimming or thickness tailoring («rhombus» or «wedge» profiles) are highly effective for frequency tuning, they introduce important aeromechanical trade-offs. Altering the blade airfoil geometry inevitably modifies the flow passage area of the inter-blade channel, flow deflection angles, and the resulting pressure distribution across the stage. Such changes can reduce the stage's aerodynamic efficiency or narrow its surge margin under severe operating conditions. Therefore, in professional engine design and overhaul practices, structural frequency shifting must not be performed in isolation; any proposed geometric refinement must be coupled with mathematical modeling using the finite element method to ensure that the aerodynamic performance and stability limits of the turbomachinery remain well within acceptable operational constraints.

4. CONCLUSION

According to the analysis results, it was determined that in order to ensure high convergence of the results of modal analysis of the blisk-type parts with the results of experimental studies, as well as simulate real operating conditions of parts at the engine operating rpm, it is necessary to take into account the centrifugal forces acting on the part and the operating temperature, while the aerodynamic flow influence on the blades can be neglected.

The issue of simplifying the geometric model of a centrifugal impeller by excluding its hub and further modal analysis of one blade requires additional research, since the hub NVFs may fall into the engine operating rpm range. A harmonic analysis makes it possible to estimate the blade and hub vibration amplitude values, as well as their impact on the performance and strength of the part, or its individual elements and, accordingly, make a decision on the possibility of simplifying the model.

When modeling the compressor axial stages, it is necessary to take into account the disk stiffness, since it significantly affects the blisk natural vibration frequencies, as well as the sequence of frequencies and natural vibration modes, although, by analogy with the

CCI, under certain conditions the model can also be simplified to determine the NVF of blades.

Changing the airfoil profile thickness («rhombus», «wedge») or trimming the edge are the effective methods for shifting the blade NVFs into a safe frequency range both during the overhaul process and in the course of manufacture of new parts. In this case, changing the blade suction surface thickness is often more effective than trimming the leading edge.

Determining the blisk modal characteristics can improve their machinability and manufacturing quality for the operations of formation of complex-profile surfaces, primarily blades.

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