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Boundary Element Method in Structural Mechanics for Periodic Structure Vibration Protection

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Abstract: *This paper investigates the adaptation of the direct boundary element method (BEM) for analyzing stationary vibrations in load-bearing structural components, specifically beams and plates, under intense dynamic loads. The study addresses the challenge of designing high-efficiency mitigation systems for engineered facilities with regular configurations, including cross-bar systems, multi-span floors, and pile foundations. A novel numerical scheme is proposed and substantiated to model bending wave propagation with high precision in periodically fixed elements. Fundamental solutions for one-dimensional and two-dimensional systems are derived using the Fourier integral transform combined with contour integration techniques. The computational advantages of the developed approach over the conventional finite element method (FEM) are demonstrated, focusing on dimensionality reduction and modeling accuracy in infinite domains, which is essential for foundation-soil interaction problems. The proposed methodology effectively identifies stop-bands-frequency ranges where wave propagation is suppressed. The findings establish a theoretical framework for designing innovative structural components capable of blocking undesirable oscillations, thereby ensuring structural safety and vibration isolation in residential and industrial environments.*

Keywords: *wave propagation; frequency stop-bands; foundation-soil interaction; numerical modeling; bending waves; vibration isolation; structural dynamics.*

1. INTRODUCTION

Modern structural mechanics imposes strict requirements on the dynamic stability of structures [1]. The occurrence of vibrations from industrial equipment, transport, or seismic impacts necessitates the development of effective vibration protection systems. Periodic structures (such as beam-and-column systems, multi-span floors, and pile foundations) possess unique wave filtration properties-specifically, the existence of so-called «stop-bands» within which vibrations do not propagate [2–4].

The use of the Boundary Element Method (BEM) offers significant advantages for vibration protection problems compared to the Finite Element Method (FEM) [5–7]. Firstly, it provides a reduction in dimensionality, as calculations are performed only on the boundary (contour) of the structure. Secondly, it ensures high accuracy in infinite domains, making BEM ideal for modeling the interaction between a foundation and the soil mass (half-space). Finally, its fundamental research approach-utilizing a fundamental solution-allows for the precise consideration of the material's dynamic characteristics [8].

The study of the dynamic response of structures to seismic and vibrational impacts remains one of the key challenges in modern structural mechanics. Objects with complex geometries and periodic structures require special attention, as local conditions and topography can significantly alter ground motion characteristics. The amplitude and frequency of seismic ground motion are critical parameters that depend on topographical irregularities.

Historically, various approaches have been used to model the processes of scattering and diffraction of seismic waves by topographic features [9]. Trifunac, along with Wong and Trifunac, investigated the displacement field over topographical objects. Boore modeled the influence of a ridge using the finite difference method, while Bouchon and Bard employed the discrete wavenumber method to simulate topographic wave diffraction [9].

Modern structural mechanics is moving away from purely deterministic models. As indicated by Wenhao Zhang, Pinghe Ni, and Mi Zhao [10], the dynamic response of underground and surface structures is significantly influenced by uncertainties in soil properties and design parameters. Several methods are used to address these issues, including the Monte Carlo Simulation (MCS), which serves as a benchmark for accuracy but requires substantial computational resources. A special place is occupied by methods based on Chebyshev series (Zhang et al. [10]), which allow for the determination of dynamic response bounds with much higher computational efficiency than MCS.

While traditional research often focused on isolated objects, modern vibration protection requires the analysis of complex «soil-structure» systems. The study of the scattering of elastic spherical P, SV, and SH waves by three-dimensional hills in a layered half-space has demonstrated that vibration amplitudes depend significantly on the spatial configuration of the landform and the specific characteristics of the soil layering [11].

For the analysis of buildings with periodic structures, the integration of wave approaches with finite element procedures is particularly effective, enabling a detailed assessment of the dynamics of metamaterials and complex engineering frames [12]. For 'nearly periodic' structures, the application of model reduction techniques based on matrix interpolation and distorted finite element meshes allows for the consideration of the cumulative effect of small uncertainties and geometric irregularities within the repeating sections of the structure [13]. In this regard, the accuracy of such dynamic simulations is highly dependent on the precise determination of the material's structural parameters. This is further addressed by Hromyak and Nemish [14], who proposed a methodology for estimating the structural parameter ρ for various engineering materials, providing a basis for refining numerical models used in the stress-strain analysis of complex systems.

The Boundary Element Method (BEM) stands out as an particularly effective tool for the dynamic analysis of both bounded and unbounded linear-elastic media [15].

Despite the extensive literature on modeling periodic systems and analyzing complex soil-structure interactions under dynamic loads, there remains a critical gap in the unified and computationally efficient formulation of direct boundary element equations specifically tailored for tracking bending wave propagation and stop-band formations in both one-dimensional (beams) and two-dimensional (plates) configurations simultaneously. Existing literature often separates these structural forms or heavily relies on computationally intensive methods that fail to naturally incorporate radiation conditions at infinity without artificial boundaries.

The objective of this study is the development of a scheme of the direct boundary element method to study the stationary vibrations of beams and the extension of this research to the analogous case for plates.

2. RESEARCH METHODOLOGY

2.1. Boundary element method formulation for one-dimensional elements (beams)

To construct a scheme for the direct boundary element method, it is necessary to determine the so-called fundamental solution of the corresponding partial differential equation (or system of equations). Physically, such a solution corresponds to the influence function. For problems in continuum mechanics, this is the displacement (or velocity) field caused by a unit concentrated force in an unbounded space. From a mathematical perspective, the fundamental solution is a solution to the corresponding differential equation where the right-hand side is a Dirac delta function.

In our case, the fundamental solution $w^*(x)$ must satisfy the following equation:

$$\frac{d^4 w^*(x)}{dx^4} - p^4 w^*(x) = -\frac{\delta}{EJ}. \tag{1}$$

To find $w^*(x)$, we use the Fourier integral transform:

$$W^*(\alpha) = \int_{-\infty}^{\infty} w^*(x) e^{-i\alpha x} dx. \tag{2}$$

As a result of application of the theorem on the differentiation of the original function, we obtain:

$$\left(\alpha^4 - p^4\right) W^*(\alpha) = -\frac{1}{EJ}. \tag{3}$$

In the derivation of (3), the fundamental integral property of the delta function was used:

$$\int_a^b f(x)\delta(x)dx = f(0), \qquad a < 0 < b, \tag{4}$$

assuming that the function itself $w^*(x)$ and its first three derivatives become infinitesimally small in magnitude as $|x| \to \infty$.

To find the solution to the algebraic equation (3) presents no difficulties:

$$W^*(\alpha) = -\frac{1}{EJ}\frac{1}{\alpha^4 - p^4}. \tag{5}$$

The most significant difficulties arise at the final stage—the stage of transitioning from the transform back to the original function:

$$w^*(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} W^*(\alpha) e^{-i\alpha x} dx = \frac{1}{4\pi p^2 EJ}\left[\int_{-\infty}^{\infty} \frac{e^{i\alpha x}}{\alpha^2 - p^2} d\alpha - \int_{-\infty}^{\infty} \frac{e^{i\alpha x}}{\alpha^2 + p^2} d\alpha\right]. \tag{6}$$

To find the second integral in the right-hand side of (6):

$$I_2 = \int_{-\infty}^{\infty} \frac{e^{i\alpha x}}{\alpha^2 + p^2} d\alpha. \tag{7}$$

for the sake of definiteness, to assume that $x > 0$.

After applying the technique of contour integration, we obtain:

$$\frac{I_{2L}}{2\pi i} = \operatorname*{Res}_{\alpha=ip} \frac{e^{i\alpha x}}{\alpha^2 + p^2} = \frac{e^{-px}}{2ip}. \tag{8}$$

Thus,

$$I_2 = \int_{-\infty}^{\infty} \frac{e^{i\alpha x}}{\alpha^2 + p^2} d\alpha = \frac{\pi}{p} e^{-px}, \qquad x > 0. \tag{9}$$

In the case when $x < 0$, to ensure that the integral over the semicircle vanishes as $R \to \infty$, the contour should be closed in the lower half-plane. Then, performing similar derivations and taking into account the presence of the pole at point $\alpha = -ip$, we obtain:

$$I_2 = \int_{-\infty}^{\infty} \frac{e^{i\alpha x}}{\alpha^2 + p^2} d\alpha = \frac{\pi}{p} e^{px}, \qquad x < 0. \tag{10}$$

Expressions (9) and (10) can be combined into one for arbitrary values of x:

$$I_2 = \int_{-\infty}^{\infty} \frac{e^{i\alpha x}}{\alpha^2 + p^2} d\alpha = \frac{\pi}{p} e^{-p|x|}. \tag{11}$$

The integrand in the first integral on the right-hand side of (6):

$$I_1 = \int_{-\infty}^{\infty} \frac{e^{i\alpha x}}{\alpha^2 - p^2} d\alpha \tag{12}$$

contains poles $\alpha = \pm p$, which lie on the real axis.

$$I_{1L} = \oint_L \frac{e^{i\alpha x}}{\alpha^2 - p^2} d\alpha = \int_{-R}^{-p+\varepsilon} \frac{e^{i\alpha x}}{\alpha^2 + p^2} d\alpha + \int_{-p-\varepsilon}^{p-\varepsilon} \frac{e^{i\alpha x}}{\alpha^2 + p^2} d\alpha + \int_{p+\varepsilon}^{R} \frac{e^{i\alpha x}}{\alpha^2 + p^2} d\alpha +$$

$$+ \int_{C_R} \frac{e^{i\alpha x}}{\alpha^2 + p^2} d\alpha + \int_{C_-} \frac{e^{i\alpha x}}{\alpha^2 + p^2} d\alpha + \int_{C_+} \frac{e^{i\alpha x}}{\alpha^2 + p^2} d\alpha \cdot \tag{13}$$

To pass that the limit in (13) as $\varepsilon \to 0$ and $R \to \infty$ and to consider that, as in the previous case, the integral over the semicircle of radius R vanishes, and there are no poles in the region bounded by the contour, we finally obtain:

$$I_1 = \int_{-\infty}^{\infty} \frac{e^{i\alpha x}}{\alpha^2 - p^2} d\alpha = -\frac{\pi}{p} \sin px, \tag{14}$$

Following similar reasoning for $x < 0$, in the general case we obtain:

$$I_1 = \int_{-\infty}^{\infty} \frac{e^{i\alpha x}}{\alpha^2 - p^2} d\alpha = -\frac{\pi}{p} \sin p|x| \cdot \tag{15}$$

Thus, the required fundamental solution is:

$$w^*(x) = \frac{1}{4p^3EJ}\left(\sin p|x| + e^{-p|x|}\right). \tag{16}$$

It is interesting to note that solution (16) is not the only possible solution to equation (1). If to introduce small internal resistance forces into the system by setting $p = -p_R - ip_I$, $p_R >> ip_I > 0$, then the contour in the bounded Fourier transform for the integral I_{1L} will completely enclose the pole $\alpha = -p_R + ip_I$, while the pole $\alpha = p_R - ip_I$ will lie outside of it.

Then, as $p_I \to 0$, to obtain:

$$w^*(x) = \frac{1}{4p^3EJ}\left(ie^{-ip|x|} + e^{-p|x|}\right). \tag{17}$$

Similarly, by setting $p = p_R + ip_I$ and then taking the limit $p_I \to 0$, to find:

$$w^*(x) = \frac{1}{4p^3EJ}\left(e^{-p|x|} - ie^{-ip|x|}\right). \tag{18}$$

All three solutions (16)–(18) are mathematically valid. Given the time dependence of all characteristics in the form $e^{-i\omega t}$, (18) corresponds to a wave caused by a unit shear force propagating to infinity from the point of application $x = 0$. Solution (17) corresponds to a wave traveling from infinity, while the «real» solution (16) is a superposition of solutions (17) and (18). It is clear that, whenever possible, it is advisable to use solution (16). However, in numerical implementation, situations sometimes arise where using (16) leads to a significant increase in calculation error. In such cases, to should resort to the physical solution (18).

Boundary integral relations, which express the values of unknown functions inside the domain through their values on the boundary (analogous to the single-layer and double-layer potentials in potential theory), are generally derived based on variational theorems such as the reciprocal work theorem. However, in this one-dimensional case, there is no need to refer to them. It is much simpler to use the relation:

$$\int_0^a\left(\frac{d^4w(x)}{dx^4} - p^4w(x)\right)w^*(x-\xi)dx = \left[\frac{d^3w(x)}{dx^3}w^*(x-\xi) - \right.$$

$$\left. - \frac{d^2w(x)}{dx^2}\frac{dw^*(x-\xi)}{dx} + \frac{dw(x)}{dx}\frac{d^2w^*(x-\xi)}{dx^2} - w(x)\frac{d^3w^*(x-\xi)}{dx^3}\right]_{x=0}^{x=a} +$$

$$+\int_0^a\left(\frac{d^4w^*(x-\xi)}{dx^4} - p^4w^*(x-\xi)\right)w(x)dx, \tag{19}$$

which is easily obtained by applying the rule of integration by parts. Here, the beam is assumed to correspond to the interval $[0,a]$ of the axis x, where $\xi \in [0,a]$.

By choosing $w^*(x)$ in the form (16), the relation (19) can be written as:

$$w(\xi)=\Big[-Q^{*}(x-\xi)w(x)+M^{*}(x-\xi)\theta(x)-\\-\theta^{*}(x-\xi)M(x)+w^{*}(x-\xi)Q(x)\Big]\Big|_{x=0}^{x=a}. \tag{20}$$

In (20), the quantities marked with an asterisk are fully determined. Taking into account the expressions for the slope angle, bending moment, and shear force according to Floquet theory [4], we find:

$$\theta^{*}(x)=\frac{1}{4p^{2}EJ}\left(\cos px-e^{-p|x|}\right)signx,\quad M^{*}(x)=\frac{1}{4p^{2}}\left(\sin p|x|-e^{-p|x|}\right),\\ Q^{*}(x)=\frac{1}{4}\left(\cos px-e^{-p|x|}\right)signx. \tag{21}$$

Among the eight quantities $w(0)$, $w(a)$, $\theta(0)$, $\theta(a)$, $M(0)$, $M(a)$, $Q(0)$ and $Q(a)$, four are specified by the boundary conditions (or four linear combinations of these quantities). Thus, in fact, only half of them are unknown. By successively taking the limits $\xi\to 0$ and $\xi\to a$, to obtain two equations for their determination. To find the other two, we differentiate (20) with respect to ξ:

$$\theta(\xi)=\left[-\frac{Q^{*}(x-\xi)}{d\xi}w(x)+\frac{M^{*}(x-\xi)}{d\xi}\theta(x)-\right.\\ \left.-\frac{\theta^{*}(x-\xi)}{d\xi}M(x)+\frac{w^{*}(x-\xi)}{d\xi}Q(x)\right]\Bigg|_{x=0}^{x=a}. \tag{22}$$

The specified two equations are again obtained using the limit transitions $\xi\to 0$ and $\xi\to a$.

The resulting system of boundary equations, which in our case are linear algebraic equations, can be represented in matrix form:

$$X_1^{T}=A_1X_1^{T}+A_2X_2^{T}. \tag{23}$$

2.2. Extension of the numerical scheme to two-dimensional elements (plates)

Following the general approach outlined above for beams, we present the main points of the direct boundary element method for solving problems of stationary vibrations of plates.

The fundamental solution of the stationary vibration equation for a plate must satisfy the following equation:

$$\Delta\Delta w^{*}(x,y)-p^{4}w^{*}(x,y)=\frac{\delta(x,y)}{D}. \tag{24}$$

To find it, we will again use the Fourier transform; as a result of its application to equation (24) and the subsequent solution of the obtained algebraic equation, we find:

$$W^*(\alpha,\beta)=\frac{1}{D}\frac{1}{\left(\alpha^2+\beta^2\right)^2-p^4}. \tag{25}$$

When restoring the original function using the inversion formula:

$$w^*(x,y)=\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}e^{i(\alpha x+\beta y)}W^*(\alpha,\beta)d\alpha d\beta \tag{26}$$

to eliminate multiple integrals, we transition to a polar coordinate system, which allows us to represent the fundamental solution in the following form:

$$w^*(x,y)=\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}e^{irp\cos(\varphi-\vartheta)}\frac{\rho d\vartheta d\rho}{\rho^4-p^4}. \tag{27}$$

Taking into account the integral representation of the Bessel function of the first kind (27) and using reasoning similar to the previous case, we obtain three possible fundamental solutions:

$$w^*(\vec{x})=-\frac{1}{4\pi p^2 D}\left[K_0(pr)-\frac{\pi i}{2}H_0^{(1)}(pr)\right],$$

$$w^*(\vec{x})=-\frac{1}{4\pi p^2 D}\left[K_0(pr)+\frac{\pi i}{2}H_0^{(2)}(pr)\right],$$

$$w^*(\vec{x})=-\frac{1}{4\pi p^2 D}\left[K_0(pr)+\frac{\pi}{2}Y_0(pr)\right], \tag{28}$$

where $H_0^{(1)}(z)=J_0(z)+iY_0(z)$ is the Hankel function of the second kind.

The first of the solutions (28) corresponds to a wave propagating to infinity, the second to a wave propagating from infinity, and the third is a superposition of the first two.

If to choose the fundamental solution of the stationary vibration equation for a plate as $w^*=w^*\left(\vec{x},\vec{\xi}\right)=w^*\left(\vec{x}-\vec{\xi}\right)$, where $\vec{\xi}=(\xi,\eta)$ is an arbitrary point in the domain Ω, then by virtue of the fundamental integral property of the delta function, the boundary integral relations will take the form:

$$w\left(\vec{\xi}\right)=\int\limits_{\Gamma}\left(-V^*\left(\vec{x},\vec{\xi}\right)w\left(\vec{x}\right)-M^*\left(\vec{x},\vec{\xi}\right)\theta\left(\vec{x}\right)+\theta^*\left(\vec{x},\vec{\xi}\right)M\left(\vec{x}\right)+\right.$$

$$\left.+w^*\left(\vec{x},\vec{\xi}\right)V\left(\vec{x}\right)\right)d\Gamma\left(\vec{x}\right)+\int\limits_{\Omega}q^*\left(\vec{x}\right)w\left(\vec{x},\vec{\xi}\right)d\Omega\left(\vec{x}\right)-$$

$$-\sum_{k=1}^{K}\overline{M}_\tau^*\left(\vec{s}_k,\vec{\xi}\right)w\left(\vec{s}_k\right)+\sum_{k=1}^{K}\overline{M}_\tau\left(\vec{s}_k\right)w^*\left(\vec{s}_k,\vec{\xi}\right). \tag{29}$$

Integral relation (29) is the basis of the direct boundary element method. It expresses the deflection inside the plate through the values of the kinematic and dynamic characteristics specified on its boundary. In its structure, it resembles a superposition of a simple potential and higher-order potentials. However, this similarity is entirely external and is explained only by the complete symmetry of the fundamental solution with respect to its arguments $\vec{x}$ and $\vec{\xi}$.

By directing the point $\vec{\xi}$ to the boundary Γ, to obtain the boundary integral equation:

$$w(\vec{\xi}) = \int_{\Gamma} \Big(-V^*(\vec{x},\vec{\xi})w(\vec{x}) - M^*(\vec{x},\vec{\xi})\theta(\vec{x}) + \\ + \theta^*(\vec{x},\vec{\xi})M(\vec{x}) + w^*(\vec{x},\vec{\xi})V(\vec{x})\Big)d\Gamma(\vec{x}). \quad (30)$$

Since only two of the four quantities w, θ, M and V are specified on the boundary Γ, to obtain a closed system of boundary integral equations (30), it must be supplemented with another relation. This can be obtained by differentiating (30) in the direction of the normal at point $\vec{\xi}$:

$$\theta(\vec{\xi}) = \int_{\Gamma} \left(-\frac{\partial V^*(\vec{x},\vec{\xi})}{\partial \vec{n}_\xi} w(\vec{x}) - \frac{\partial M^*(\vec{x},\vec{\xi})}{\partial \vec{n}_\xi} \theta(\vec{x}) + \\ + \frac{\partial \theta^*(\vec{x},\vec{\xi})}{\partial \vec{n}_\xi} M(\vec{x}) + \frac{\partial w^*(\vec{x},\vec{\xi})}{\partial \vec{n}_\xi} V(\vec{x}) \right) d\Gamma(\vec{x}). \quad (31)$$

If to direct $\vec{\xi}$ to the boundary Γ, then (30) and (31) will constitute a system of boundary integral equations for the theory of stationary vibrations of plates. In the case of a simple domain geometry, the system (30) – (31) can be investigated analytically.

3. ANALYSIS OF RESULTS AND DISCUSSION

3.1. Discretization of boundary integral equations

The continuous boundary integral equations derived in the previous section for plates are transformed into a computationally implementable numerical model using the boundary element method discretization procedure. The boundary Γ of the plate is divided into a finite number N of boundary elements. Within each element, the unknown physical variables-specifically the deflection, normal slope, bending moment, and effective shear force-are assumed to be constant and equal to their values at the midpoints of the respective elements.

As a result of this spatial discretization, the continuous boundary integral equations are successfully reduced to the following global system of linear algebraic equations:

$$w(\vec{x}_i) = \sum_{j=1}^{N} \left[A_{ij} w(\vec{x}_j) + B_{ij}\theta_n(\vec{x}_j) + C_{ij} M_n(\vec{x}_j) + D_{ij} V_n(\vec{x}_j) \right],$$

$$\theta_n(\vec{x}_i) = \sum_{j=1}^{N} \left[E_{ij} w(\vec{x}_j) + F_{ij}\theta_n(\vec{x}_j) + G_{ij} M_n(\vec{x}_j) + H_{ij} V_n(\vec{x}_j) \right], \quad (32)$$

$$i = 1,2,\ldots,N,$$

where N is the number of boundary elements

3.2. Calculation of off-diagonal and diagonal self-influence coefficients

To calculate the coefficients in (32), let us introduce some notation. For this to transition from the global system (x, y) to a local coordinate system associated with the j -th element (Fig. 1).

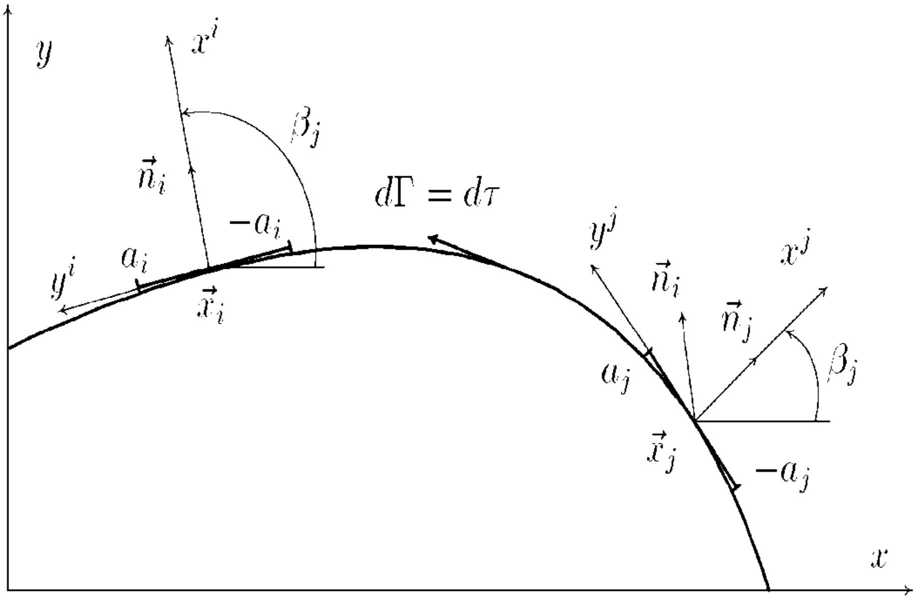

Figure 1. Local coordinate systems associated with boundary elements.

After cumbersome but straightforward transformations, to find the expressions for the off-diagonal coefficients $(i \neq j)$:

$$A_{ij} = \frac{p}{4\pi} x_i^j \int_{-a_j}^{a_j} \left(Z_3(pr) - 4\frac{Z_2(pr)}{pr} \right) \frac{dy^j}{r} - \frac{1-\nu}{4\pi} x_i^j Z_2(pr) \frac{y^j - y_i^j}{r^2} \Bigg|_{y^j=-a_j}^{y^j=a_j},$$

$$B_{ij} = \frac{1}{4\pi} \int_{-a_j}^{a_j} \frac{Z_2(pr)}{r^2} \left(\left(x_i^j\right)^2 + \nu\left(y^j - y_i^j\right)^2 \right) dy^j + \frac{(1+\nu)D}{x_i^j} C_{ij},$$

$$C_{ij} = -\frac{x_i^j}{4\pi p D} \int_{-a_j}^{a_j} \frac{Z_1(pr)}{r} dy^j, \qquad D_{ij} = \frac{1}{4\pi p^2 D} \int_{-a_j}^{a_j} Z_0(pr) dy^j,$$

$$E_{ij} = \frac{p}{4\pi} \left[n_1 \left\{ p \int_{-a_j}^{a_j} \left[Z_4(pr) \frac{\left(x_i^j\right)^2}{r^2} - Z_3(pr) \frac{r^2 + 6\left(x_i^j\right)^2}{pr^3} + Z_2(pr) \frac{4}{p^2 r^2} \right] dy^j - \right.\right.$$

$$\left. - (1-\nu) \left[\left(Z_3(pr) \frac{\left(x_i^j\right)^2}{r^2} - \frac{Z_2(pr)}{pr} \right) \frac{y^j - y_i^j}{r} \right] \Bigg|_{y^j=-a_j}^{y^j=a_j} \right\} + \qquad (33)$$

$$+n_2\left\{\left[Z_3(pr)-4\frac{Z_2(pr)}{pr}+(1-\nu)\left(Z_3(pr)\left(\frac{y^j-y_i^j}{r}\right)^2-\frac{Z_2(pr)}{pr}\right)\right]\frac{x_i^j}{r}\right\}\Bigg|_{y^j=-a_j}^{y^j=a_j},$$

$$F_{ij}=\frac{1}{4\pi}\left[pn_1\int_{-a_j}^{a_j}\left(Z_3(pr)\frac{\left(x_i^j\right)^2+\nu\left(y^j-y_i^j\right)^2}{r^2}-(3+\nu)\frac{Z_2(pr)}{pr}\right)\frac{x_i^j}{r}dy^j+\right.$$

$$\left.+n_2\left(\frac{Z_2(pr)}{r^2}\left(\left(x_i^j\right)^2+\nu\left(y^j-y_i^j\right)^2\right)-(1-\nu)\frac{Z_1(pr)}{pr}\right)\Bigg|_{y^j=-a_j}^{y^j=a_j}\right],$$

$$G_{ij}=-\frac{n_1}{4\pi D}\int_{-a_j}^{a_j}\left(Z_2(pr)\frac{\left(x_i^j\right)^2}{r^2}-\frac{Z_1(pr)}{pr}\right)dy^j-\frac{n_2}{4\pi pD}Z_1(pr)\frac{x_i^j}{r}\Bigg|_{y^j=-a_j}^{y^j=a_j},$$

$$H_{ij}=-C_{ij}n_1+\frac{Z_0(pr)}{4\pi p^2D}n_2\Bigg|_{y^j=-a_j}^{y^j=a_j},$$

where a_j is the half-length of the j – th boundary element, $Z_n(t)=\frac{\pi i}{2}H_n^{(1)}-K_n(t)$, $r=\sqrt{\left(y^j-y_i^j\right)^2+\left(x_i^j\right)^2}$.

The five diagonal coefficients (self-influence coefficients) are determined directly from (33) by setting що $x_i^j=y_i^j=0$, $n_1=1$, $n_2=0$, then:

$$B_{jj}=-\frac{1}{2\pi p}\left[\frac{\pi i}{2}I_H(pa_j)+I_K(pa_j)-(1-\nu)Z_1(pa_j)\right],\quad C_{jj}=0,$$

$$D_{jj}=-\frac{1}{2\pi p^3D}\left[\frac{\pi i}{2}I_H(pa_j)-I_K(pa_j)\right], \tag{34}$$

$$G_{jj}=-\frac{1}{2\pi pD}\left[\frac{\pi i}{2}I_H(pa_j)+I_K(pa_j)-Z_1(pa_j)\right],\quad H_{jj}=0,$$

where $I_H(x)=\int_0^x H_0^{(1)}(x)dx$, $\qquad I_K(x)=\int_0^x K_0(x)dx$.

The other three coefficients are expressed via singular integrals, which should be understood as the Cauchy principal value. The final result of such an asymptotic analysis can be presented in the form:

$$A_{jj}=F_{jj}=1/2,\quad E_{jj}=\frac{p}{2\pi}\left[\int_0^{pa_j}Z_0(x)dx\frac{\pi i}{2}+Z_3(pa_j)-(3+\nu)\frac{Z_2(pr)}{pa_j}\right]. \tag{35}$$

The expressions for the coefficients (33) – (35), together with the system of linear algebraic equations (32), form the basis of the numerical scheme for solving boundary value problems in the theory of stationary vibrations of plates. The deflection values at the internal points of the plate, after solving of this system, are found using the appropriately discretized relation (30).

The numerical scheme developed in this study demonstrates high computational efficiency and mathematical rigor in analyzing the stationary vibrations of periodically constrained beams and plates. By utilizing the direct Boundary Element Method (BEM) combined with Fourier transform and contour integration techniques, the proposed method achieves a reduction in spatial dimensionality, which significantly lowers the computational cost compared to traditional domain-discretization techniques like the Finite Element Method (FEM). This advantage becomes particularly pronounced when modeling infinite or semi-infinite domains, such as foundation-soil interactions, where BEM naturally satisfies the radiation conditions at infinity without requiring artificial absorbing boundaries.

The analytical determination of the diagonal self-influence coefficients by executing a limiting transition ensures numerical stability and precision. Resolving the singular integrals in the sense of the Cauchy principal value prevents the typical matrix ill-conditioning issues often encountered in classic boundary element formulations when dealing with complex structural geometries or high-frequency ranges. The practical value of the presented model lies in its ability to accurately identify and track frequency stop-bands in periodic structural systems, providing engineers with a reliable tool for designing effective passive vibration protection barriers and foundations under intense dynamic or seismic loads.

Constraints and limitations. Despite its advantages, the proposed methodology has certain limitations that outline directions for future research. Currently, the formulated numerical scheme is strictly applicable to linear-elastic media and thin structural elements governed by the Kirchhoff-Love plate theory and Euler-Bernoulli beam theory. Thick plates or structures under extreme cyclic loads involving material non-linearity, plastic deformations, or viscoelastic damping effects are beyond the scope of the present formulation. Furthermore, the fundamental solutions derived here assume ideal, perfectly rigid periodic boundary constraints. In real-world engineering scenarios, geometric irregularities, structural uncertainties, or soil-structure interface degradation could disrupt perfect periodicity, which may lead to wave localization effects that require stochastic or modified boundary element approaches.

4. CONCLUSIONS

The developed BEM numerical scheme is a powerful tool for structural mechanics. Unlike classical methods, it allows for high-precision modeling of dynamic processes in vibration protection elements, accounting for wave radiation energy and the specific characteristics of periodic systems in construction.

The proposed method allows for:

- determination of the optimal periodicity steps of supports to create «wave barriers» within structures;
- evaluation of the reduction level of vibration accelerations in residential premises as waves propagate through the building frame;
- designing of construction metamaterials — structures capable of completely blocking specific frequency ranges of manufactured noise.

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