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Mathematical Models of Electrocardiosignals for the Task of Their Simulation, Taking into Account Various Types of Artifacts

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Abstract: *This article presents the developed new stochastic mathematical models of the electrocardiosignal, which allow to take into account the randomness and cyclicity of its physical nature and the possibility of taking into account various types of artifacts, which may be due to various reasons (zero drift of the isopotential line, poor contact of the electrodes with the patient's body surface, etc.). The artifacts that allow to take into account mathematical models primarily include those that manifest themselves in a change in the shape of the electrocardiosignal, that is, atypical changes in morphology not associated with signs of pathologies or signs of a normal signal. These atypical changes are taken into account by mathematical models both on cycle segments and on individual segments-zones of the electrocardiosignal. In addition, the developed mathematical models allow to take into account changes in the rhythm of such a cyclic signal by taking it into account in the rhythm function, and therefore, in addition to artifacts that manifest themselves in the morphology of the signal, mathematical models allow to lay down possible artifacts associated with the rhythm during modeling.*

Keywords: *electrocardiosignal, cyclic random process, random process, mathematical model, computer modeling, segmentation, cycle segments, zone segments, processing methods, statistical processing*

1. INTRODUCTION

The modern development of information technologies is opening up increasing opportunities for the creation of advanced diagnostic systems that enable highly accurate and objective diagnostics. At the same time, an important task arises, which consists in testing such newly developed diagnostic systems in order to determine their effectiveness and the challenges associated with processing data using new methods, which, in turn, are based on novel mathematical models.

Such testing is possible through the use of two approaches. The first involves the use of open databases of real diagnostic signals, such as PhysioNet, MIT-BIH Arrhythmia Database, European ST-T Database, TB Diagnostic ECG Database, and others. The second approach involves the use of computer simulation tools for cyclic signals, which make it possible to imitate various test electrocardiosignals (ECG). These signals incorporate informative features related to rhythm and morphology. It is precisely these features that newly developed methods in the tested diagnostic systems are expected to detect and respond to. In practice real cyclic signals may include artifacts. Artifacts are non-informative components of a signal whose occurrence is not related to the activity of the heart muscle. They are caused by various factors, such as poor contact between the electrode

and the patient's body surface, and others. Such artifacts require identification and either removal or exclusion from the corresponding cycles or diagnostic segments during ECG processing in order to ensure correct analysis and diagnosis.

A significant number of open databases of diagnostic electrocardiosignals do not address the subtask of generating ECG signals that simultaneously contain a specific pathology – manifested either in rhythm [1] or in morphology – while also including artifacts. For example, such artifacts may be caused by baseline wander in the second ECG cycle or by poor electrode contact appearing in the third cycle, among others.

Therefore, the development of mathematical models that enable the creation of computer-based ECG simulation methods – while embedding information about the type of pathology and possible types of artifacts into the simulated signal – constitutes a relevant scientific task.

1.1 Problem statement

This publication is devoted to the development of new mathematical models for the problem of computer simulation of electrocardiosignal realizations, which make it possible to account for artifacts of different physical nature within the structure of a cyclic signal.

1.2. Analysis of recent research and publications

The electrocardiosignal has a complex temporal structure and is characterized by the presence of repetitive cycles, each of which consists of diagnostic zones: P, Q, R, S, T (commonly used in medical practice). These zones reflect the cyclic activity of the cardiac muscle. In a statistical sense, such a signal can be considered a non-stationary random process whose parameters change over time in accordance with the cardiac cycle [2]. At the same time, under real recording conditions, the electrocardiosignal is often distorted by various types of noise and artifacts caused by external electromagnetic fields, patient movements, instability of electrode contact, muscle activity, and other factors. The presence of such artifacts complicates the signal analysis process and may lead to reduced accuracy of diagnostic algorithms.

In the direction of developing mathematical models for ECG simulation, there are many approaches, and three main directions can be identified:

- *Physiological models* provide sufficiently high accuracy in simulating the morphological features of the signal but have limited ability to incorporate the diversity of both morphological and rhythmic characteristics. These models are mainly used to describe a single cardiac cycle of the ECG, often at the level of cells or tissues (e.g., membrane potential models-physiological models) [3, 4].

Such models are based on physiological and biophysical descriptions of cardiac activity, for example, models of cardiomyocyte action potential. They ensure high accuracy of morphological features of the signal, such as the P, QRS, and T waves, and comply with the physiological laws of heart function. Because of this, they are often used to study cardiac mechanisms and to model specific pathologies. Alongside these advantages, these models also have drawbacks related to limited variability of morphological and rhythmic patterns, difficulty in scaling to large datasets, and the need for detailed physiological information about the heart.

- Statistical models allow the simulation of both individual elements of the cardiac cycle and specific pathologies [2,5]. These models can incorporate information about the rhythm of the cyclic signal; however, most of them only account for a constant rhythm, and thus do not allow modeling of different types of arrhythmias.

This modeling approach uses parametric or stochastic models to represent individual elements of the cardiac cycle or an entire ECG realization. Some models allow for incorporating variations in heart rhythm as well as phases of the cardiac cycle (diagnostic zones). These models are less computationally expensive and are well suited for generating ECG fragments and can be used for training classification algorithms. The disadvantages of this approach include lower morphological accuracy which is lower than that of physiological models, possible simplifications or generalizations in the model, and such models do not fully reflect the biophysical processes of heart function.

- Deep generative models (GAN – Generative Adversarial Networks, diffusion). This line of model development provides the greatest diversity of simulated signals used for augmenting medical datasets; however, the correctness of their operation largely depends on the training process, the chosen generative models, and the input signals used for training [6].

This approach to model construction ensures the widest variety of simulated (artificial, synthetic) signals and is actively used to expand medical datasets. These models are capable of generating ECG signals with various pathological variations and complex morphological structures that are difficult to obtain using traditional methods. Significant drawbacks include the lack of guarantees of physiological plausibility, strong dependence on the volume and quality of input data, issues with training stability (especially for GANs), high computational costs, and difficulty in controlling signal morphology. Synthetic signals require thorough validation before clinical or scientific application.

Let us consider in more detail recent studies and works that use the approaches described above.

In [7], a model for ECG signal generation is proposed that accounts for time-varying signal parameters and enables the simulation of different types of arrhythmias. This model employs a discrete Markov chain to describe transitions between normal and abnormal rhythms (arrhythmias). It also allows modeling the dependence of QRS complex morphology and PQ and QT intervals on heart rate. However, the model does not fully account for the structural information of ECG signals and possible types of artifacts.

In recent years, an increasing number of studies have focused on the use of deep generative models for generating (modeling) so-called synthetic (artificial) cardiac signals. In [6], a comparison of several GAN-based models for generating synthetic cardiac signals was conducted. The authors demonstrated that generative models can produce cardiac cycles with morphology close to real signals and can be used to balance medical datasets for subsequent processing stages. They also proposed metrics for evaluating the quality of generated signals, including Dynamic Time Warping, Fréchet distance, and Euclidean distance. The use of such models makes it possible to synthesize rare pathological heart conditions (for example, certain types of arrhythmias [8]), which is important for training artificial intelligence systems.

In [9], a model for ECG generation based on a combination of recurrent neural networks (RNN – Recurrent Neural Network) and generative adversarial networks (GAN) is presented.

In [10], a multi-level generation model is proposed, which first models individual cardiac cycles and then constructs long time sequences while accounting for dependencies between heartbeats. This approach allows the generation of simulated ECG signals lasting several minutes, with rhythm and morphological characteristics close to real ECGs. However, this model does not incorporate the ability to account for artifacts [11]. An advantage of the

proposed approach is the ability to consider cardiac cycles of varying durations, which is important for modeling arrhythmias.

In works [12, 13], the concept of constructing generative adversarial networks (GANs), widely used for generating artificial signals, is presented. In [14], the use of a modified GAN architecture for synthetic ECG generation is proposed. In [15], an approach to generating electrocardiographic signals based on diffusion models (DM – Diffusion Models) is introduced.

There are also studies based on differential equations that are used for reconstructing ECG signals from photoplethysmographic (PPG) signals. For example, in [16], a method for ECG generation using generative networks is proposed, where simulated cardiac signals are used to train neural network models.

Statistical approaches to the development of mathematical models and ECG signal processing methods are discussed in works [2, 5, 17], which describe signal processing techniques and statistical methods for ECG analysis.

The existence of various approaches to ECG modeling indicates that not all problems of computer-based electrocardiographic signal modeling have been fully solved. Therefore, the development of mathematical models for such tasks remains highly important.

2. Review of Mathematical Models for Computer Simulation of ECG Signals Considering Artifacts

The analysis of publications shows that electrocardiogram (ECG) signal modeling is actively developing and is based on a combination of various mathematical models, machine learning methods, and generative neural networks. In recent years, the most widely used approaches include modern techniques such as GAN models and diffusion models (DMs), which enable the generation of ECG signals close to real ones, including various pathological conditions and artifacts. Such approaches contribute to the development and testing of automated medical diagnostic systems and the creation of new methods for biomedical signal analysis.

However, these approaches have several drawbacks. In particular, the use of GAN models is associated with training complexity, and their effectiveness largely depends on the correctness of training and the diversity of input signals used for training. Another significant limitation of this approach is the inability to explicitly incorporate the required informational features of rhythm, morphology, and artifacts into the modeled signal in a controlled manner.

The use of diffusion models implies high computational complexity in their implementation algorithms. These models also have a strong dependence on the input data required for training, which may result in generated data that significantly differ from real signals and may contain uncontrolled artifacts (i.e., those not intentionally introduced by the researcher in the modeling process). In this case, such signals do not guarantee physiological correctness. This is particularly critical for medical signals, as pathological cases are often rare and insufficient for complete model training.

A major drawback of diffusion models is their high computational cost. These computationally intensive models require significant GPU/TPU resources and a long time for training and signal generation, which is a substantial limitation, especially for integration into real medical systems or for rapid prototyping in scientific research.

The use of GAN models is also limited by difficulties in controlling signal properties. It is sometimes impossible to ensure realistic signals while precisely controlling specific characteristics of synthetic signals (heart rate, amplitude, shape of P, QRS, and T waves). This makes them less flexible compared to physiological or statistical models. At the same time, the risk of errors and spontaneous (uncontrolled) artifacts increases, which may be imperceptible to humans but can distort the training of automatic classifiers or diagnostic algorithms.

3. Mathematical Model

When initiating the development of a mathematical model, a set of requirements for it should be formulated.

First, the mathematical model must account for cyclicity caused by the activity of the heart.

Second, it should be stochastic and incorporate randomness, since in this case it will better reflect the variability of such a biological system.

Third, it should allow the incorporation of both variable rhythm (normal conditions and pathological cases such as arrhythmias) and a constant rhythm (for example, one induced by a pacemaker).

Fourth, it must account for the structure of cycles composed of segment-zones (diagnostic zones).

Fifth, it should be capable of representing artifacts that manifest in morphology (within cycles and/or zones) as well as in rhythm.

One of the mathematical models that satisfies most of the above requirements is the cyclic random process [17, 18]. For such a model, statistical processing methods have been developed that allow estimation of statistical characteristics of both morphological and rhythmic signal features, and these can be used for further modeling steps.

Let us define a cyclic random process as a separable stochastic process $\xi(\omega,t), \omega\in\mathbf{\Omega}, t\in[0,T)$, which is called a cyclic random process with a continuous argument if there exists a function $T(t,n)$ satisfying the conditions of a rhythm function [18], such that the finite-dimensional vectors $(\xi(\omega,t_1), \xi(\omega,t_2),\ldots,\xi(\omega,t_k))$, and $(\xi(\omega,t_1+T(t_1,n)), \xi(\omega,t_2+T(t_2,n))),\ldots,\xi(\omega,t_k+T(t_k,n))$, $n\in\mathbf{Z}$, where $\{t_1, t_2,\ldots,t_k\}$ is the separability set of the process $\xi(\omega,t), \omega\in\mathbf{\Omega}, t\in[0,T)$, are stochastically equivalent is in the wide sense for all integers. $k\in\mathbf{N}$.

An important characteristic of this mathematical model is the rhythm function $T(t,n)$. It is a function that describes the law of variation of time intervals between its in-phase values. The main properties of this function are described in [18]. It allows one to account for the rhythm of a cyclic ECG as either constant or variable.

However, this mathematical model does not allow artifacts within cycles or diagnostic zones to be taken into account. Therefore, new mathematical models will be developed based on it.

Considering the substantiated mathematical model as a base model based on a cyclic random process, we will first examine a developed mathematical model that accounts for artifacts within an entire segment-cycle, and then separately consider artifacts within diagnostic segment-zones.

For this purpose, let us examine in more detail the structure of the mathematical model that accounts for segment-cycles. To this end, we represent the mathematical model of a realization of a cyclic random ECG process in a form that reflects its segmented cyclic structure as follows [18].

$$\xi_\omega(t)=\sum_{i=1}^{C} f_i(t), t\in\mathbf{W}, \qquad (1)$$

where C is the number of segment-cycles in the realization of the cyclic random ECG process. $\mathbf{W}$ is the domain of definition of the cyclic ECG process, and its range, in the case of a

stochastic approach, is a Hilbert space of random variables defined on a probability space $\left(\xi_\omega(t) \in \mathbf{\Psi} = \mathbf{L}_2(\mathbf{\Omega}, \mathbf{P})\right)$.

In this mathematical model, the segment-cycles of the cyclic process are defined using indicator functions:

$$f_i(t) = \xi_\omega(t) \cdot I_{\mathbf{W}_i}(t), i = \overline{1, C},\ t \in \mathbf{W}. \quad (2)$$

In this case, the indicator functions that single out the cycle segments are defined as follows:

$$I_{\mathbf{W}_i}(t) = \begin{cases} 1, t \in \mathbf{W}_i, \\ 0, t \notin \mathbf{W}_i. \end{cases}, \quad (3)$$

where $\mathbf{W}_i$ is the domain of the indicator function (for the cycle segments), which in the case of a discrete signal, i.e. $\mathbf{W} = \mathbf{D}$, is equal to a discrete set of samples:

$$\mathbf{W}_i = \left\{t_{i,l}, l = \overline{1, L}\right\}, i = \overline{1, C}, \quad (4)$$

The segmental cyclic structure is represented by a set of time samples $\{t_i\}$ or $\left\{t_{i,l}\right\}$, $i = \overline{1, C}$, $l = \overline{1, L}$. In this formulation of the mathematical model (1), the rhythm of the cyclic ECG process is taken into account via a continuous rhythm function $T(t, n)$, namely:

$$I_{\mathbf{W}_i}(t) = I_{\mathbf{W}_{i+n}}\left(t + T(t, n)\right),\ i = \overline{1, C}, n \in \mathbf{Z}, t \in \mathbf{W}. \quad (5)$$

Since the rhythm in the mathematical model is taken into account through the inclusion of the rhythm function $T(t, n)$. then, at the stage of ECG processing, it is first necessary to determine the segmental cyclic structure, i.e. to find the set $\hat{\mathbf{D}}_c = \{t_i, i = \overline{1, C}\}$, which represents the set of time instants corresponding to the boundaries of the cycle segments of the ECG process [18].

Let us consider, within this mathematical model, the case where the ECG realization contains a cycle segment with an artifact, denoted by $\tilde{f}_i(t)$; for example, let this be the 2nd cycle. ($i = 2$):

$$\tilde{f}_i(t) = \tilde{\xi}_\omega(t) \cdot I_{\mathbf{W}_i}(t),\ i = 2, \quad (6)$$

where $\tilde{\xi}_\omega(t)$ is a realization that represents a certain type of artifact on the cycle segments.

Let us introduce two types of sets $\mathbf{S}$, and $\mathbf{A}$ which do not intersect, i.e.:

$$\mathbf{S} \cap \mathbf{A} = \varnothing, \quad (7)$$

Where the set $\mathbf{S}$ corresponds to the indices of cycle segments that do not contain artifacts, and the set $\mathbf{A}$ contains the indices of cycle segments in which an artifact is observed.

In this case, the union of these sets yields the set $\mathbf{C}$ which contains all elements (cycle indices) of the analyzed ECG realization, $i=\overline{1,C}$, $C=5$ (for the given example):

$$\mathbf{S} \bigcup \mathbf{A} = \mathbf{C}. \tag{8}$$

For this example, the realization of the ECG consists of 5 cycles, i.e. $C=5$, $i=\overline{1,5}$. As noted above, the second cycle ($i=2$) contains an artifact. Then the sets $\mathbf{S}$, $\mathbf{A}$ will contain the following elements $\mathbf{S}=\{1,3,4,5\}$, $\mathbf{A}=\{2\}$, and the set will be the union of the sets $\mathbf{S}$ and $\mathbf{A}$.

Now let us introduce the coefficients s_i, a_i $i=\overline{1,C}$, which will serve as «keys» for including or excluding the artifact in the corresponding i-th segment-cycle in the new mathematical mod.

$$s_i = \begin{cases} 1, i=\mathbf{S}, \\ 0, i\neq\mathbf{S} \end{cases}, i=\overline{1,C} \tag{9}$$

$$a_i = \begin{cases} 1, i=\mathbf{A}, \\ 0, i\neq\mathbf{A} \end{cases}, i=\overline{1,C} \tag{10}$$

Let us take into account the introduced coefficients s_i, a_i as well as the sets $\mathbf{S}$ and $\mathbf{A}$, in formula (2), rewriting it as follows for our example:

$$\begin{aligned} \xi_{art\omega}(t) = \xi_{\omega}(t)\cdot I_{\mathbf{W}_1}(t)\cdot s_1 + \tilde{\xi}_{\omega}(t)\cdot I_{\mathbf{W}_1}(t)\cdot a_2 + \\ + \xi_{\omega}(t)\cdot I_{\mathbf{W}_3}(t)\cdot s_3 + \xi_{\omega}(t)\cdot I_{\mathbf{W}_4}(t)\cdot s_4 + \xi_{\omega}(t)\cdot I_{\mathbf{W}_5}(t)\cdot s_5, \end{aligned} \tag{11}$$

where $\xi_{art\omega}(t)$ is an ECG realization that contains an artifact in the 2nd cycle segment.

For formula (11), the key coefficients s_i, a_i $i=\overline{1,C}$, will be equal to $s_i=\{1,0,1,1,1\}$, $a_i=\{0,1,0,0,0\}$, $i=\overline{1,C}$, $C=5$.

Let us write the mathematical model in a generalized form, taking into account artifacts in the segment-cycles:

$$\xi_{art\omega}(t) = \sum_{i=1}^{C}\left(f_i(t)\cdot s_i + \tilde{f}_i(t)\cdot a_i\right), \tag{12}$$

where $f_i(t)$ are the segment-cycles of the ECG, and $\tilde{f}_i(t)$ are the segment-cycles that contain artifacts.

Let us write the mathematical model that accounts for artifacts in any segment-cycle (a generalized form of the mathematical model using indicator functions).

$$\xi_{art\omega}(t)=\sum_{i=1}^{C} I_{\mathbf{W}_i}(t)\left(\xi_{\omega}(t)\cdot s_i+\tilde{\xi}_{\omega}(t)\cdot a_i\right),\ i=\overline{1,C}\,. \tag{13}$$

The structural diagram of the method for computer modeling of ECS, which allows accounting for artifacts in segment-cycles, is shown in Figure 1.

In this structural diagram, the block «Modeling of ECG realizations without artifacts» uses the computer modeling method described in [17]. To account for morphological features (morphological characteristics) in the simulated ECG realizations, statistical estimates of the mathematical expectation $\hat{m}_{\xi_{T(t,n)}}(t)$, and variance $\hat{d}_{\xi_{T(t,n)}}(t)$ obtained from processing real ECG data are used.

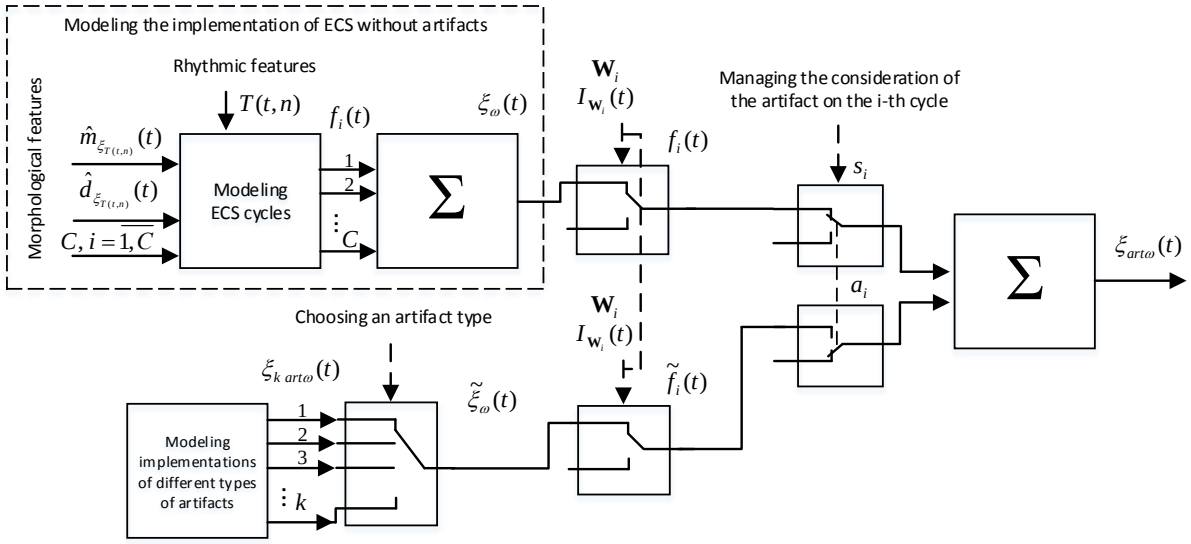

Figure 1. Structural diagram of computer simulation of ECG with consideration of artifacts in segment-cycles

Statistical estimates can be defined by formulas [17]:

Estimation of the mathematical expectation:

$$\hat{m}_{\xi_{T(t,n)}}(t)=\frac{1}{C}\sum_{n=1}^{C}\xi_{\omega}\left(t+T(t,n)\right),\ t\in\mathbf{W}_1=\left[t_1,t_2\right), \tag{14}$$

where t_1,t_2 are discrete time samples corresponding to the start and end of the first cycle segment, and C is the number of cycles.

Estimation of the variance:

$$\hat{d}_{\xi_{T(t,n)}}(t)=\frac{1}{C}\cdot\sum_{n=1}^{C}\left[\xi_{\omega}\left(t+T(t,n)\right)-\hat{m}_{\xi_{T(t,n)}}(t)\right]^2,\ t\in\mathbf{W}_1=\left[t_1,t_2\right). \tag{15}$$

To incorporate rhythm features (rhythmic characteristics) into the modeled realization, a rhythm function $T(t,n)$ is used in this block.

The block for modeling realizations of various types of artifacts is not considered in this work; the mathematical model for modeling artifact realizations will be presented in a subsequent study.

In this structural diagram, indicator functions $I_{\mathbf{W}_i}(t)$ are used, which make it possible to isolate segment-cycles in the modeled ECG realization without artifacts $\xi_\omega(t)$ and in the modeled realization of a selected type of artifact $\tilde{\xi}_\omega(t)$, by taking into account the domain of definition of the i -th cycle $\mathbf{W}_i$. The keys s_i, a_i $i=\overline{1,C}$ are used to account for the artifact in the selected corresponding segment-cycle.

Since within ECG cycles there are diagnostic regions (segment-zones), we will consider the developed mathematical model that accounts for artifacts in segment-zones. For this purpose, let us consider the mathematical model of ECG that takes into account the zonal-segment structure [18].

$$\xi_\omega(t)=\sum_{i=1}^{C} f_i(t)=\sum_{i=1}^{C}\sum_{j=1}^{Z} f_{i_j}(t),\ t\in\mathbf{W} \tag{16}$$

In this mathematical model, the zone segments of the cyclic process are defined using indicator functions, i.e.:

$$f_{i_j}(t)=\xi_\omega(t)\cdot I_{\mathbf{W}_{i_j}}(t)=f_i(t)\cdot I_{\mathbf{W}_{i_j}}(t),\ i=\overline{1,C},\ j=\overline{1,Z},\ t\in\mathbf{W} \tag{17}$$

In this case, the indicator functions that isolate the zone segments are defined as follows:

$$I_{\mathbf{W}_{i_j}}(t)=\begin{cases}1,\ t\in\mathbf{W}_{i_j},\\ 0,\ t\notin\mathbf{W}_{i_j}.,\end{cases} \tag{18}$$

where $\mathbf{W}_{i_j}$ is the domain of the indicator function (for zone segments), which, in the case of a discrete signal, i.e. $\mathbf{W}=\mathbf{D}$, corresponds to a discrete set of samples.

$$\mathbf{W}_{i_j}=\left\{t_{i_{j,l}},l=\overline{1,L_j}\right\},\ L=\sum_{j=1}^{Z}L_j, \tag{19}$$

In such a formulation of the mathematical model (17), the rhythm of the cyclic ECG process is also taken into account through a continuous rhythm function $T(t,n)$, namely:

$$I_{\mathbf{W}_{i_j}}(t)=I_{\mathbf{W}_{i+n_j}}\left(t+T(t,n)\right),\ i=\overline{1,C},\ j=\overline{1,Z},\ n\in\mathbf{Z},\ t\in\mathbf{W} \tag{20}$$

The rhythm in this mathematical model is taken into account through the rhythm function $T(t,n)$. In order to determine the rhythm function, it is first necessary to define the segmental zone structure, i.e. to find $\hat{\mathbf{D}}_z=\{t_{i_j},i=\overline{1,C},j=\overline{1,Z}\}$, which represents the set of time instants corresponding to the boundaries of the ECG zone segments.

Thus, having determined the segmental cyclic structure, i.e. finding $\hat{\mathbf{D}}_c=\{l_i,i=\overline{1,C}\}$, or the segmental zone structure $\hat{\mathbf{D}}_z=\{t_{i_j},i=\overline{1,C},j=\overline{1,Z}\}$ it is possible to estimate the

rhythmic structure for the segmental cyclic structure $\left\{\hat{T}(t_i, n),\ i=\overline{1,C}, n\in\mathbf{Z}\right\}$ and for the segmental zone structure $\{\hat{T}(t_{i_j}, n),\ i=\overline{1,C},\ j=\overline{1,Z}, n\in\mathbf{Z}\}$ respectively [18].

Let us consider the case when the ECG consists, for example, of 5 cycles, i.e. $C=5$, $i=\overline{1,5}$, but each cycle contains 5 zones, i.e. $Z=5$, $j=\overline{1,5}$. As noted above, in the considered example, the second cycle ($i=2$), contains an artifact; however, this time it is not present over the entire cycle, but only in zones 4 and 5, i.e. ($j=4,5$).

Then the sets $\mathbf{S}$, and $\mathbf{A}$, which correspond to the cycle segments (containing cycle indices), will contain the following elements $\mathbf{S}=\{1,3,4,5\}$, $\mathbf{A}=\{2\}$ and the set $\mathbf{C}=\{1,3,4,5,2\}$.

Since, in the case of accounting for finer segments than cycles (zone segments) in the mathematical model, it is important to know not only the cycle number in which the artifact is observed but also the zone segment number, we introduce the notation of sets $\mathbf{S}_i$ and $\mathbf{A}_i$ which indicate the corresponding cycle $i=\overline{1,C}$ and contain elements (zone segment indices) $j=\overline{1,Z}$. In analogy with formulas (7) and (8), we obtain:

$$\mathbf{S}_i \bigcup \mathbf{A}_i = \mathbf{C}_i,\ \mathbf{S}_i \bigcap \mathbf{A}_i = \varnothing,\ i=\overline{1,C}, \tag{21}$$

where $\mathbf{S}_i$ is the set corresponding to the indices of zone segments that do not contain artifacts, and $\mathbf{A}_i$ is the set corresponding to the indices of zone segments that contain an artifact in the corresponding i-th cycle ($i=2$) and j-th zone segment ($j=4,5$) of the ECG realization.

By analogy with formulas (9) and (10), we introduce coefficients s_{i_j}, a_{i_j} $i=\overline{1,C}$, $j=\overline{1,Z}$ which will act as «keys» for including or excluding the artifact in the corresponding i-th segment-cycle and the corresponding j-th segment-zone.

$$s_{i_j} = \begin{cases} 1, i=\mathbf{S}_i, \\ 0, i\neq\mathbf{S}_i \end{cases}, i=\overline{1,C},\ j=\overline{1,Z}, \tag{22}$$

$$a_{i_j} = \begin{cases} 1, i=\mathbf{A}_i, \\ 0, i\neq\mathbf{A}_i \end{cases}, i=\overline{1,C},\ j=\overline{1,Z}. \tag{23}$$

Now let us take into account the introduced coefficients and sets in formula (16), rewriting it to account for artifacts in the corresponding segment-zones as follows:

$$\xi_{art\omega}(t)=\sum_{j=1}^{5}\xi_{\omega}(t)\cdot I_{\mathbf{W}_{1_j}}(t)\cdot s_{1_j}+\sum_{j=1}^{3}\xi_{\omega}(t)\cdot I_{\mathbf{W}_{2_j}}(t)\cdot s_{2_j}+$$

$$\tilde{\xi}_{\omega}(t)\cdot I_{\mathbf{W}_{2_4}}(t)\cdot a_{2_4}+\tilde{\xi}_{\omega}(t)\cdot I_{\mathbf{W}_{2_5}}(t)\cdot a_{2_5}+\sum_{i=3}^{5}\sum_{j=1}^{5}\xi_{\omega}(t)\cdot I_{\mathbf{W}_{i_j}}(t)\cdot s_{i_j}, \quad (24)$$

where $\xi_{art\omega}(t)$ is an ECG realization that contains an artifact in the 2nd segment-cycle and in the 4th and 5th segment-zones.

Let us consider why the sets $\mathbf{S}_i$, $\mathbf{A}_i$ are defined as such for the given example in the case of the second segment-cycle ($i=2$). The sets will contain the following elements (indices of segment-zones): $\mathbf{S}_2=\{1,2,3\}$, $\mathbf{A}_2=\{4,5\}$, while the set $\mathbf{C}_2=\{1,2,3,4,5\}$ will be the union of the sets $\mathbf{S}_2$ and $\mathbf{A}_2$ and will contain all elements (indices of segment-zones).

Let us write the generalized mathematical model that accounts for artifacts in any segment-cycle and segment-zone in the following form (generalized form of the mathematical model) by incorporating indicator functions.

$$\xi_{art\omega}(t)=\sum_{i=1}^{C}\sum_{j=1}^{Z}I_{\mathbf{W}_{i_j}}(t)\left(\xi_{\omega}(t)\cdot s_{i_j}+\tilde{\xi}_{\omega}(t)\cdot a_{i_j}\right) \quad (25)$$

For formula (24), taking into account the values of the key coefficients s_{i_j}, a_{i_j} for the segment-cycle ($i=2$) that contains artifacts in the segment-zones ($j=4,5$), they will be equal to $s_{2_4}=\{1,1,1,0,1\}$, $a_{2_4}=\{0,0,0,1,0\}$, $s_{2_5}=\{1,1,1,1,0\}$, $a_{2_5}=\{0,0,0,0,1\}$, for the remaining segment-cycles and segment-zones that do not contain artifacts, they will be equal to $s_{i_j}=\{1,1,1,1,1\}$, $a_{i_j}=\{0,0,0,0,0\}$, $i=1,3,4,5$, $C=5$, $j=\overline{1,5}$, $Z=5$.

Let us present the generalized mathematical model that accounts for artifacts in segment-cycles and segment-zones:

$$\xi_{art\omega}(t)=\sum_{i=1}^{C}\sum_{j=1}^{Z}f_{i_j}(t)\cdot s_{i_j}+\tilde{f}_{i_j}(t)\cdot a_{i_j}. \quad (26)$$

The structural diagram of the computer simulation method for ECG, which allows accounting for artifacts in segment-zones, is shown in Figure 2.

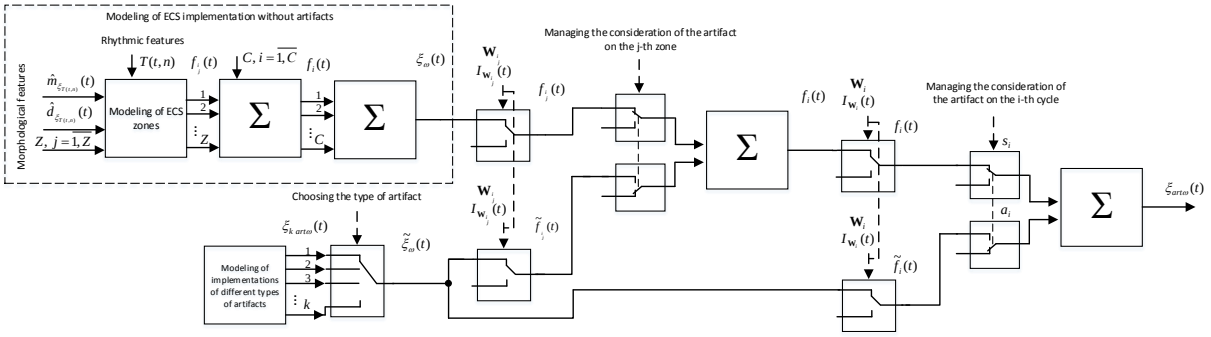

Figure 2. Structural diagram of computer simulation of ECG with consideration of artifacts in segment-zones and segment-cycles

$I_{\mathbf{W}_{i_j}}(t)$

In this structural diagram, indicator functions are used, which allow the selection of segment-zones in the modeled ECG realization without artifacts and in the modeled realization of a selected type of artifact, by taking into account the domain of definition of the i -th segment-cycle $\mathbf{w}_i$ and the j -th segment-zone $\mathbf{W}_{i_j}$. The keys s_{i_j}, a_{i_j} $i=\overline{1,C}$, $j=\overline{1,Z}$ are used to account for the artifact in the corresponding segment-zone.

4. CONCLUSIONS

This work develops two stochastic mathematical models that make it possible to account for artifacts in segment-cycles and segment-zones, as well as to perform computer modeling of ECG. In addition, the mathematical models allow separate incorporation of information about morphology, both under normal conditions and for various types of pathologies, in the modeled ECG realizations at the level of segment-cycles and diagnostic segment-zones.

Furthermore, the models make it possible to incorporate information about variable or constant rhythm by accounting for a rhythm function within the mathematical framework. The study also presents structural schemes of methods for computer modeling of ECG realizations with consideration of artifacts, corresponding to the developed mathematical models.

Future research will involve computer modeling based on the developed mathematical models, as well as a detailed description of a newly developed mathematical model specifically intended for modeling realizations of various types of artifacts.

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