Scientific Journal of the Ternopil National Technical University
2026, vol. 122, no. 2, pp. 11–21,
ISSN: 2522-4433 Print, ISSN: 2522-4441 Online
https://doi.org/10.35784/ visnyk_tntu2026.02.011

Submitted: 2026-01-27 | Revised: 2026-03-05 | Accepted: 2026-05-26

Equivalent Axisymmetric 2D Modeling of Three-Dimensional Stress State

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Abstract: *The study addresses the urgent problem of rapid structural integrity assessment for VVER-1000 reactor pressure vessels (RPVs) under pressurized thermal shock (PTS) conditions during lifetime extension. Given the high computational complexity of three-dimensional finite element modeling, the authors propose a fast computational scheme based on 2D axisymmetric models. The paper analyzes the effect of stiffness mismatch between the simplified models and the actual geometry of nozzle zones, and proposes a combined model that merges the exact vertical profile of the nozzle with the corresponding RPV shell segment. The proposed technique enables the transformation of stresses from the 2D formulation into complete profiles across the entire wall thickness, ensuring the accuracy of stress intensity factor (SIF) calculations at the level of full-scale 3D models while significantly reducing computational time.*

Keywords: *reactor pressure vessel, VVER-1000, pressurized thermal shock, lifetime extension, 2D axisymmetric modeling, stress intensity factor, structural integrity.*

1. INTRODUCTION

The lifetime extension of operating nuclear power plant units is a strategic objective for the energy sector, requiring justification of the reliability and safe operation of critical equipment. The reactor pressure vessel is one of the most critical components, the condition of which determines the overall service life of the reactor plant. According to industry standards [1 and 2], a computational assessment of brittle fracture resistance is a mandatory requirement for demonstrating RPV structural integrity both during its design lifetime and during its extension.

The most critical among the transients are the regimes referred to in specialized literature as pressurized thermal shock. This process is characterized by a rapid drop in temperature on the inner surface of the RPV wall while maintaining high operating pressure. Such a combination of thermal shock and mechanical loading leads to high tensile stresses, creating a risk of initiation and unstable growth of potential or existing defects.

The primary safety criterion in fracture toughness assessments is the determination of the allowable temperature margin, T_{ka}. This indicator is defined as the minimum difference between the current metal temperature and the critical brittle fracture temperature. The finite element method (FEM) is the traditional tool for determining the stress-strain state and the SIF.

However, performing serial three-dimensional calculations for complex transient regimes with a large number of time steps is extremely time-consuming. This creates a critical need for rapid assessment methods.

For the rapid determination of K_I, the influence coefficient method or the weight function method is widely used in international practice (API 579-1/ASME FFS-1 [3] and ASME BPVC Section XI [4]). Their primary advantage is the ability to determine the SIF based solely on the active stress profile in an unflawed model.

Modern advancements in this field focus on establishing a 'computational bridge' between complex FEM simulations and applied algorithms. This is evidenced by the developments of Boz and Demir [5], who utilize FCPAS-type systems to transform numerical data arrays into enhanced empirical equations for nozzle zones. Parallel to this, analytical methods proposed by Xue et al. [6] enable the description of the complete stress field in the intersection zone of two cylinders, while Changsik et al. [7], [8] have further advanced these approaches through algorithms that transform stresses from the cylindrical shell portion into complete profiles across the entire thickness of the nozzle wall.

The engineering analysis of stress-strain states in complex structures is based on a decomposition methodology that relies on analytical solutions for regular structural sections. The calculation begins by determining the nominal stresses in the cylindrical section using elasticity theory. To transition to the local stress state in stress concentration zones, these basic values are adapted using stress concentration factors. The effectiveness of the approach has been confirmed in calculations of superheater headers [9] and the VVER-1000 reactor core barrel [10], where the basic stress fields served as the basis for evaluating fracture mechanics parameters.

The challenges of applying 2D axisymmetric models have been analyzed in detail in the works of D. Sommerville and M. Walter [11]–[14]. The authors established that the primary obstacle is the stiffness mismatch between the 2D model and the actual 3D geometry, which leads to errors in determining hoop stresses. To mitigate this effect, the use of correction factors and the calibration of simplified models against reference 3D calculations were substantiated.

The key issue with using simplified axisymmetric models for the analysis of RPV nozzle zones is the significant mismatch in stiffness between the structural components. Under internal pressure, the cylindrical shell portion of the vessel undergoes substantial radial and hoop deformations, whereas the nozzle zone, due to its increased wall thickness, exhibits much higher stiffness. Consequently, the nozzle acts as a constraining factor, restricting the free expansion of the shell. A standard axisymmetric model is inherently incapable of fully accounting for these spatial constraints and the influence of the overall vessel stiffness on the nozzle zone.

In view of this, the present work proposes approaches that enable the calculation of the stress-strain state based on rapid 2D models with an accuracy comparable to full-scale 3D modeling. Figure 1 shows the full three-dimensional model of the RPV nozzle along with the potential axisymmetric options:

Model A represents the nozzle profile in the plane aligned with the symmetry axes of the reactor pressure vessel and the main coolant pipeline (MCP);

Model B is based on the RPV cross-section passing through the nozzle axis perpendicular to the longitudinal axis of the vessel;

Model C (combined) is developed to mitigate the significant geometric differences between the previous schemes and the actual vertical profile at the location of the postulated defect. It combines the exact profile of the nozzle in the vertical section with the corresponding segment of the RPV shell, which maximizes the alignment of the calculation conditions with the real geometry in the defect zone.

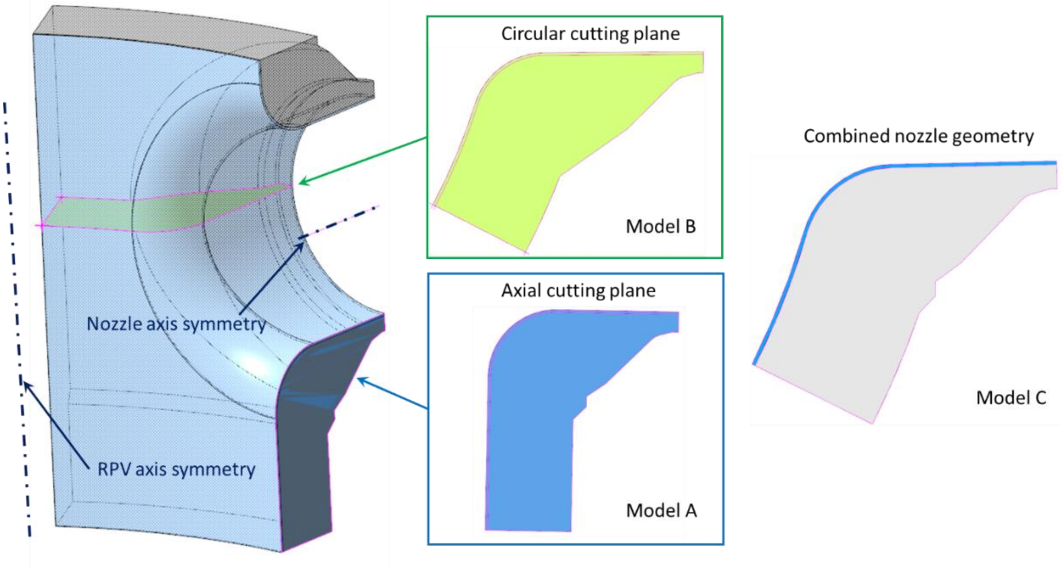


Figure 1. Geometric models of the RPV nozzle.

The proposed methodology allows for maintaining the high computational speed characteristic of 2D problems, while simultaneously compensating for the lack of spatial constraints through boundary conditions optimization and the use of combined geometry.

2. METHODS

The present work implements two independent approaches aimed at verifying the results of two-dimensional finite element modeling and aligning them with the three-dimensional stress-strain state in the nozzle joint zone.

2.1. Stress field transformation using influence functions

The first approach is based on applying an influence function used as a correction operator to establish a correspondence between the stress-strain state obtained in the 2D formulation and the spatial stress distribution in the actual 3D FE model of the RPV nozzle. The influence function compensates for the simplifications associated with the transition from a 3D to a 2D model (see Figure 2). It enables integrating the actual stiffness of the 3D structure and the deformation constraints at the nozzle-to-vessel junction, which cannot be reproduced solely through planar geometry.

To construct this function, an approach based on comparing the results of 2D and 3D finite element modeling is used. The stress state obtained from the spatial 3D model of the nozzle is taken as the reference; it is considered the baseline, accounting for the three-dimensional structural behavior of the reactor pressure vessel. Based on the corresponding stress values obtained for the postulated defect zone, the following relation is derived:

$$R(t) = \frac{\sigma_{3D}(t)}{\sigma_{2D}(t)} \quad (1)$$

where: $\sigma_{3D}(t)$ – is the stress obtained from the three-dimensional model,
$\sigma_{2D}(t)$ – is the corresponding stress from the two-dimensional formulation,
t – is the coordinate or parameter along the control path, which represents a normal from the center of the nozzle blend radius in this work.

The obtained spatial distribution of the coefficient R(x) is used to approximate the influence function F(x), which is subsequently applied as a correction operator to the results of 2D modeling. Thus, the influence function is derived based on a numerical comparison of stress distributions in 2D and 3D formulations and reflects the generalized effect of three-dimensional geometry and the loss of volumetric deformation constraints in the simplified model. The formula for obtaining equivalent spatial stresses is as follows:

$$\sigma_{3D}(t) = \sigma_{2D}(t) \cdot F(t) \quad (2)$$

In addition, an adaptive approach to determining the transformation function was used to calculate stresses caused by temperature differences. In this adaptive approach, the stress correction $\Delta\sigma(x)$ is defined as the product of the temperature difference ΔT between the outer and inner surfaces of the wall and the transformation function with respect to thickness.

$$\Delta\sigma(t) = \Delta T \cdot \mathrm{F}(t) \quad (3)$$

Accordingly, the final function for transforming 2D stresses into 3D takes the following form:

$$\sigma_{3D}(t) = \sigma_{2D}(t) + \Delta T \cdot \mathrm{F}(t) \quad (4)$$

2.2. The method of equivalent boundary conditions

The equivalent forces method is an effective tool for engineering analysis. It enables the simulation of complex spatial interactions among elements within simplified computational models. Its essence lies in compensating for lost geometric connections by iteratively selecting load parameters. This ensures the physical validity of the stress distribution in the nozzle zone at minimal computational cost. The application of this method allows 2D models to be adapted to the actual operating conditions of the heat exchanger while maintaining the necessary accuracy of the results.

A key element of the methodology is the application of equivalent forces that approximate the stress distribution of the simplified model to that of a three-dimensional model. In this case, we apply internal pressure and tensile forces to the end face of the RPV shell. The combination of end face distortion limitation and a compensating force factor allows us to:

- Quantitatively – align the absolute values of hoop stresses in 2D and 3D formulations;
- Qualitatively – reproduce the characteristic stress gradient in the defect postulation zone, which arises due to the spatial interaction of structural elements.

The application of this method ensures a high degree of agreement between the results and reference three-dimensional calculations, while significantly reducing computational costs, which is critical for performing probabilistic failure analysis or fracture mechanics analyses.

2.3. Computational models and boundary conditions

FE Model A is used to implement the stress field transformation method via influence functions (see Figure 2). To ensure the static determinacy of the model, a Frictionless Support constraint is applied to the nozzle end face (on the MCP side). In the axisymmetric formulation, this fixes axial displacements, which is a necessary and sufficient condition for the kinematic stabilization of the calculation scheme. At the same time, the absence of constraining boundary conditions on the shell end face leads to the distortion of its cross-section. Therefore, to ensure the physical accuracy of the deformation and to prevent cross-sectional distortion, a node degree-of-freedom coupling algorithm was used. In the ANSYS software package, this condition is realized through the Coupling mechanism [15], initiated by the APDL command: CP, 1, UY, ALL. This imposes a kinematic constraint on all nodes of the shell end face, forcing them to displace uniformly along the Y-axis (the RPV axis). This approach implements the plane-sections hypothesis, which allows nodes to be freely distributed across the end-face plane without restricting the model's expansion.

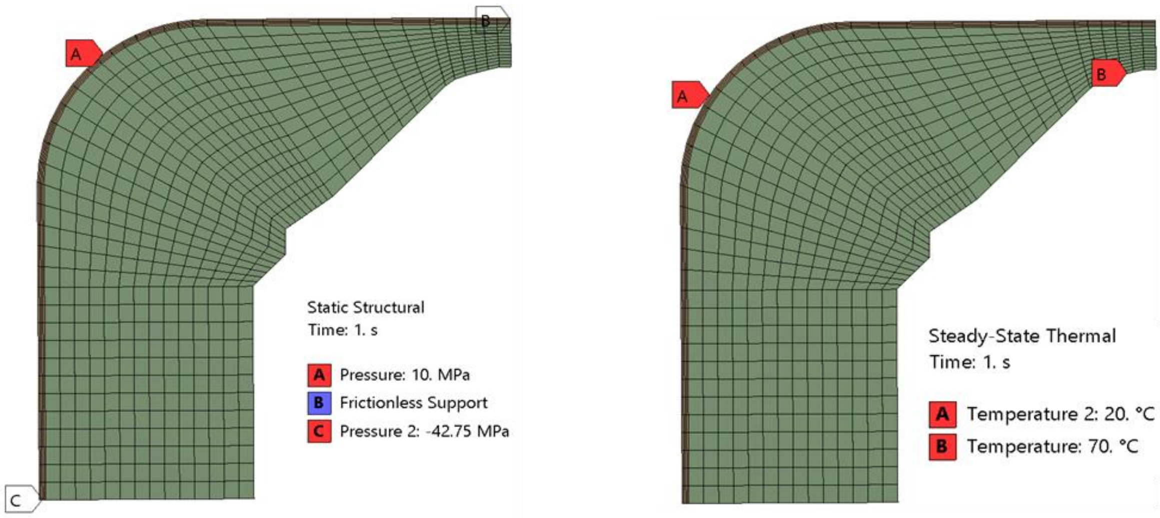


Figure 2. Finite element model and applied boundary conditions for the stress transformation approach.

In addition to the kinematic boundary conditions, the structural problem incorporates internal pressure and a tensile force applied to the shell end face, whereas the thermal problem involves temperature loads applied to the inner and outer wall surfaces.

To implement the equivalent boundary conditions method, FE Model A was utilized for calculating the stress-strain state under internal pressure. In solving the thermal problem and determining the stresses caused by the temperature gradient, the combined Model C was used to achieve maximum agreement with the 3D modeling results. This model includes a portion of the cylindrical shell, which allowed for the correct replication of thermal inertia and the kinematic constraints of the junction without artificial adjustment of heat fluxes. A schematic representation of the FE models and the adopted boundary conditions is shown in Figure 3.

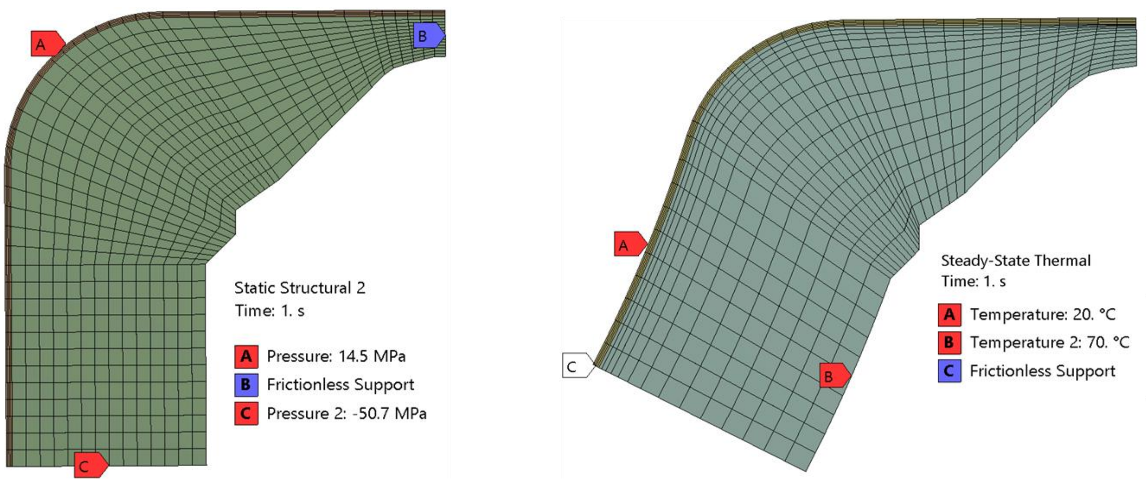


Figure 3. Finite element models and applied boundary conditions for the equivalent load approach.

Model A uses the same kinematic boundary conditions described earlier, plus equivalent forces. Model C includes a shell segment, so its boundary conditions change: a Frictionless Support is applied to the shell end face, while the Coupling mechanism is used for the MCP end face. For the thermal problem, both models use the exact same temperature boundary conditions as the previous method.

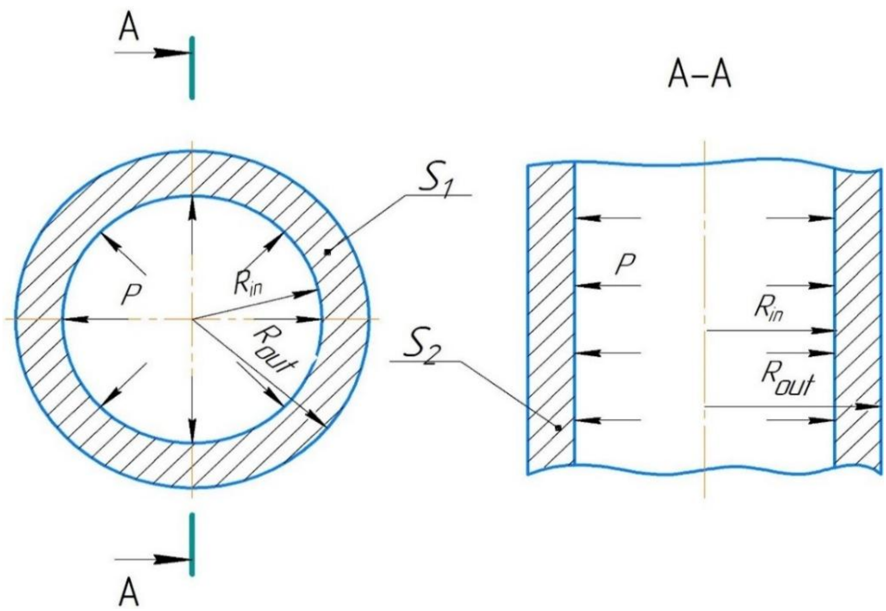


Figure 4. Geometric parameters.

Figure 4 shows the RPV cross-sections in the circumferential and axial directions. Based on equations (3) and (4), the compensating force was calculated and applied to the surfaces as an equivalent pressure.

For the RPV nozzle shell the values of $\mathrm{R_{in}} = 1990\mathrm{mm}$ and $\mathrm{R_{out}} = 2285\mathrm{mm}$ are $\mathrm{K_1} = 3.14$ and $\mathrm{K_2} = 6.746$, respectively.

$$P_{ax} = \mathrm{P}K_1 = P\frac{R_{in}^2}{R_{out}^2 - R_{in}^2} \tag{5}$$

$$P_{cr} = \mathrm{P}K_2 = P\frac{R_{in}}{R_{out} - R_{in}} \tag{6}$$

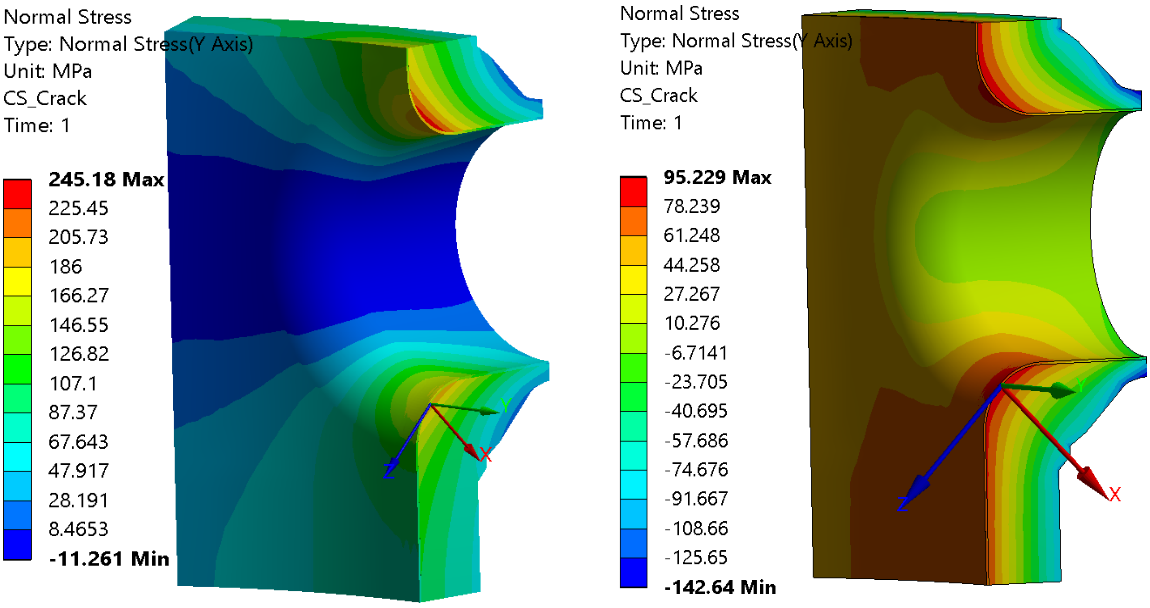
We used an average value for the coefficient $\mathrm{K_{avg}} = 4.943$ because axisymmetric models do not allow separation of boundary conditions into axial and circumferential components. When rotated around the axis of symmetry, the model end faces describe a cylindrical surface whose projection areas onto the planes corresponding to the axial and circumferential directions are identical. Therefore, the force contributions from both directions are equal, making the use of the arithmetic mean coefficient physically justified and correct.

3. RESULTS

To construct stress transformation functions for the RPV nozzle zone, the results of the 3D model calculation are used as reference data. These results were obtained for two types of baseline unit loads:

- An internal pressure of 10 MPa is applied to the internal surfaces of the model;

- A thermal load in the form of a linear temperature gradient across the RPV wall thickness with a temperature differential of 50 °C.



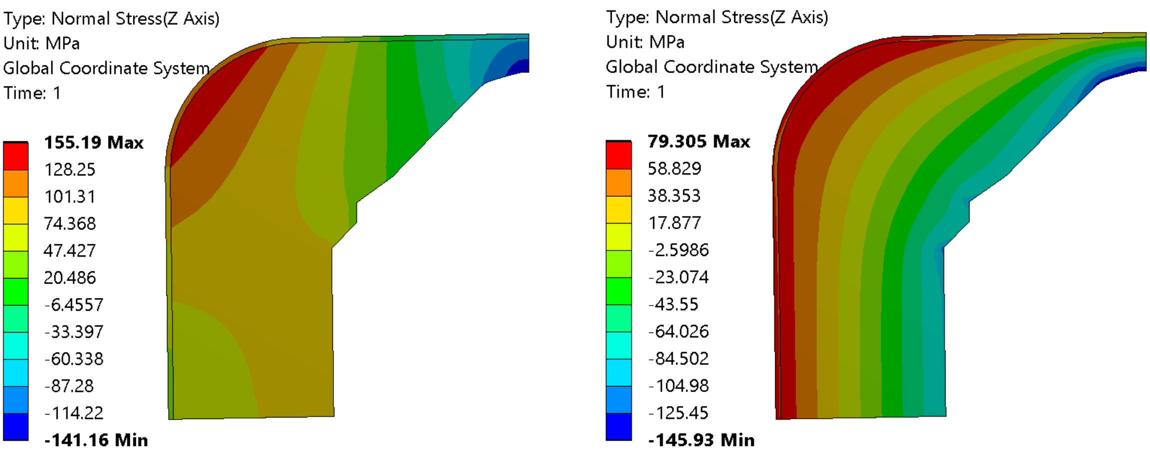
(a) (b)

Figure 5. Reference results.

Figure 5 presents the stress fields for the specified load cases. The stress distribution under internal pressure is labeled as (a), while the results corresponding to the thermal load are shown under (b).

3.1. Stress transformation function approach

The stress-strain state of the RPV nozzle zone is shaped by the combined effects of mechanical and thermal loads. Therefore, to develop the stress transformation function approach, we analyzed each of these load cases separately. To implement this approach, a 2D axisymmetric model lying in the RPV plane of symmetry was utilized. Figure 6 presents the stress fields due to both internal pressure and a linear temperature gradient.



(a) (b)

Figure 6. Initial 2D simulation results for the stress transformation function approach.

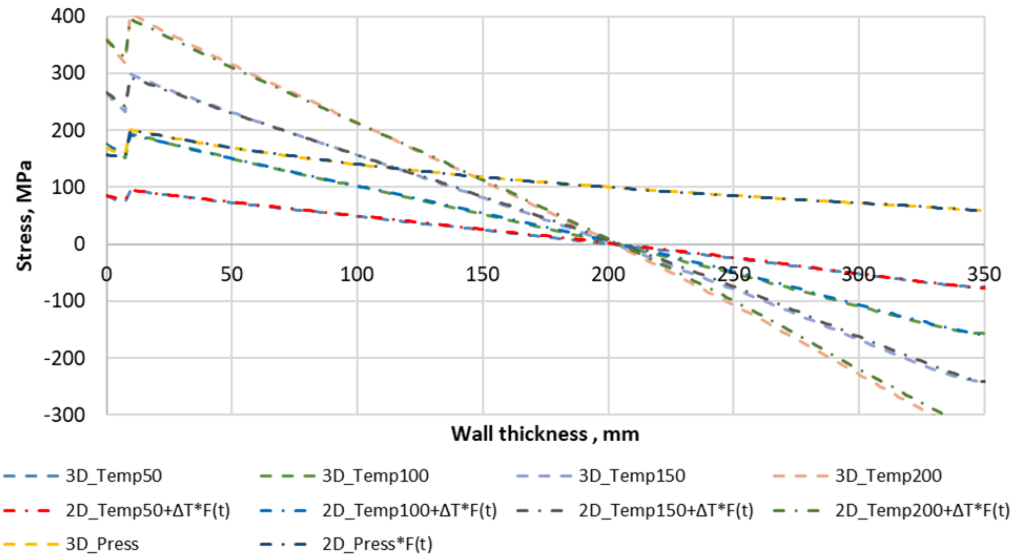


Figure 7. Hoop stress distribution obtained via the stress transformation function approach

Figure 7 presents a comparison of the stress distributions obtained using the 3D spatial and 2D axisymmetric models for the baseline load cases. The thermal shock scenario occurs with varying temperature gradients across the wall thickness. Therefore, the graph displays the calculation results for temperature differences of 50, 100, 150, and 200 °C between the outer and inner walls. The 'Press' and 'Temp' labels refer to the internal pressure and thermoelasticity problems, respectively. The dashed lines correspond to the baseline loading, while the dash-dotted lines represent the stresses from the 2D model corrected via the transformation function.

The presented results demonstrate that the proposed transformation functions perform effectively for both pressure loading and thermal loading problems. Unlike the pressure load cases, the temperature distribution exhibits a more complex non-linear behavior. Consequently, applying a basic multiplicative function without accounting for the temperature gradient across the wall thickness led to significant errors.

The determination of the stress transformation coefficient is based on a direct comparison of the results from the 3D spatial and 2D axisymmetric models. Since the stress distribution is similar in both cases, this approach allows deriving a transformation function that compensates for the quantitative discrepancy between them.

Figure 8 presents the variation of this coefficient across the wall thickness. As illustrated by the plot, the coefficient is not constant, which is attributed to the specific geometry of the blending radius and the stress concentration captured in the 3D formulation.

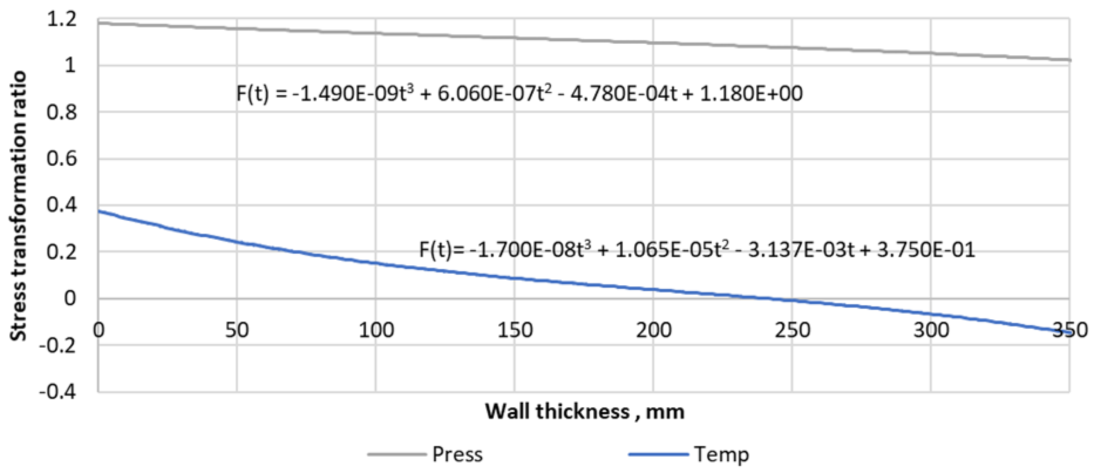


Figure 8. Variation of the transformation coefficient.

The introduction of stress transformation coefficients allows for the complete elimination of these discrepancies. The alignment of the refined curves 2d_Press*F(t) and 2D_Temp* F(t) (see Figure 7). The reference 3D results clearly demonstrate the effectiveness of the proposed transformation method. It is evident that the variation of the transformation coefficient across the wall thickness, shown in Figure **8**, reflects the functional relationship between the 2D and 3D models. Consequently, the application of these coefficients ensures an accurate transition to refined stress-strain state values while maintaining the high computational speed of the axisymmetric models.

3.2. Equivalent load approach

To compensate for the absence of the RPV shell influence, the equivalent load approach was applied. Through iterative adjustment and comparison with the reference 3D results, it was determined that for a nominal pressure of 10 MPa, the equivalent internal pressure must be increased to 14.5 MPa, combined with a tensile stress of 50.7 MPa applied to the free end of the nozzle. This specific combination qualitatively reproduces the «stretching» effect exerted by the reactor shell on the nozzle. Figure 9 shows the stress distribution in the axisymmetric models. As shown in Figure 10, the 2D model closely replicates the stress-strain state of the 3D FE model, with the hoop stresses in the defect zone underestimated by only 2%.

When analyzing the stress-strain state induced by a thermal gradient, the choice of the boundary conditions is dictated by the critical need to account for real heat transfer conditions. Although the thermal gradient can be artificially increased in test cases to achieve a target stress level, the physical validity of the model is compromised by this approach.

In brittle fracture resistance calculations, the target parameters include not only SIF but also the local temperature within the defect zone, as the material's fracture toughness KIc directly depends on temperature. Consequently, any methodology that directly manipulates the temperature profile to compensate for geometric simplifications is methodologically unacceptable, as it distorts the temperature field and compromises the validity of the brittle fracture strength assessment.

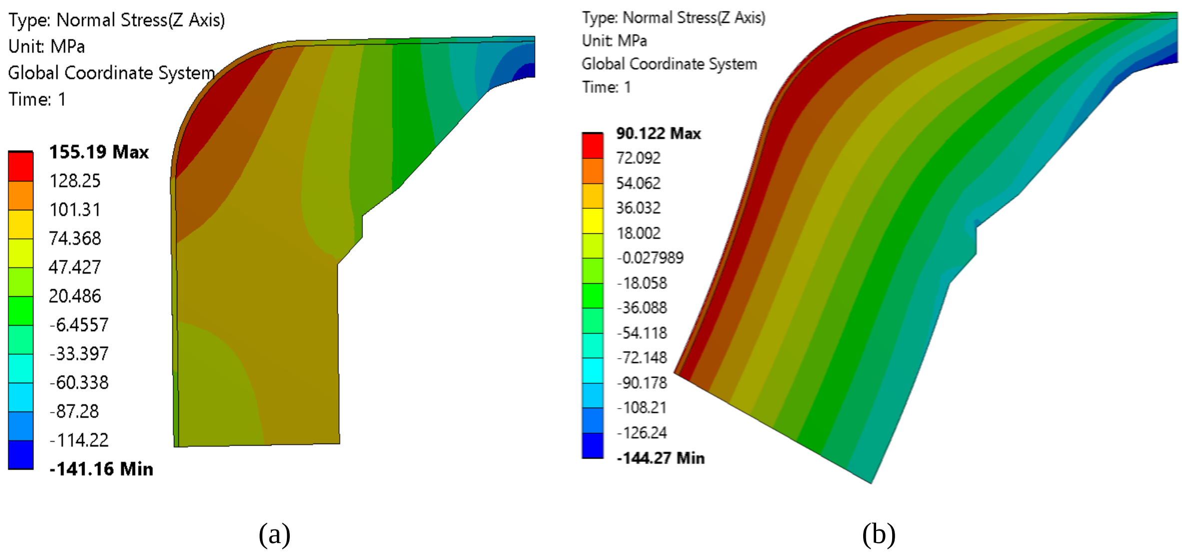


Figure 9. 2D FEM simulation results obtained by the equivalent load approach.

Figure 10 presents the verification of the results, confirming the high efficiency of the chosen approach. As shown, the 2D stresses qualitatively and quantitatively match those of the 3D model. The maximum error at the postulated defect location is 5%.

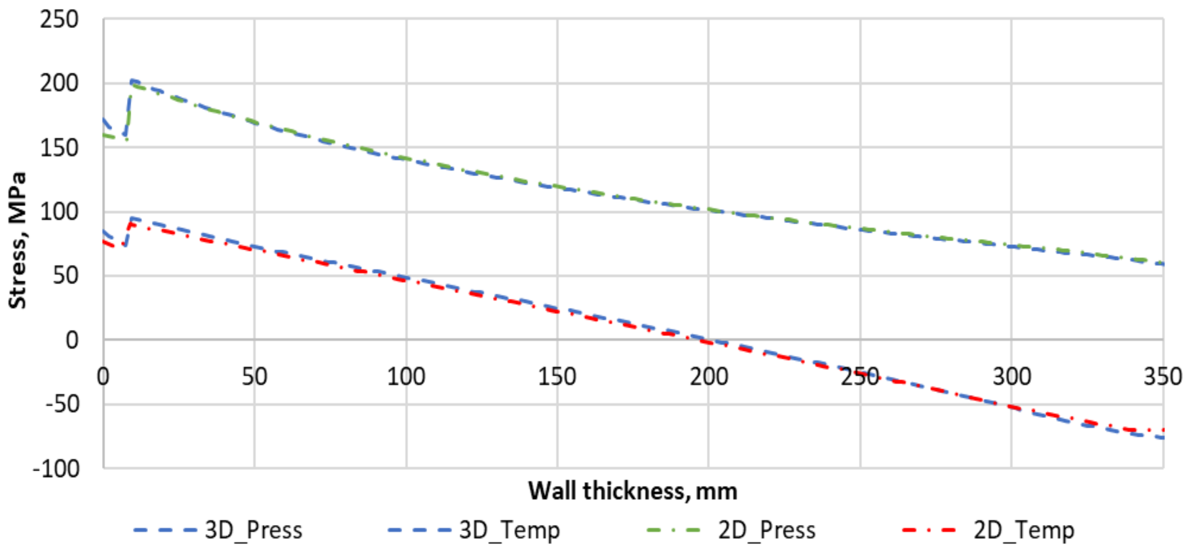


Figure 10. Hoop stress distribution obtained via the equivalent load approach.

4. DISCUSSION

The application of a stress transformation function that varies across the wall thickness yields significantly higher accuracy than the conventional approach [11–14], which adjusts the entire stress profile using a single peak factor. Using a constant coefficient leads to excessive conservatism, whereas the proposed method accounts for the actual through-wall stress gradient.

At the same time, implementing a combined model alongside the equivalent boundary conditions method represents a more sophisticated solution than attempting a direct adjustment of the 2D model. This avoids the need for rigorous monitoring of all complex physical processes during thermal transients, as the model provides a more physically accurate representation of structural stiffness and its deformation response to thermal shock.

The need for a detailed nozzle model is driven by the complex boundary conditions of the transient thermal problem. These complex thermal-hydraulic conditions preclude the application of simplified stress transformation approaches that rely solely on scaling solutions derived from the cylindrical shell components.

The application of an axisymmetric approach within the NPP equipment assessment ensures a significant reduction in computation time, which is key for modeling long-term transient processes. Despite the geometric simplification, this approach allows for high accuracy in calculating stress fields necessary for the precise evaluation of fracture mechanics parameters. Furthermore, reducing the problem dimensionality enables the use of significantly denser finite element meshes in stress concentration zones, particularly around the nozzle blend radius. Finally, this minimization of computational costs provides an opportunity for efficient serial multi-parametric calculations.

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