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Algorithm for developing a mechanical and technological model of cleaning systems for root harvesting machines

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Abstract: *The development and optimization of the parameters of new working bodies and machines generally requires a phased analysis and generalization of existing technologies, methods of harvesting root crops, design features of modern technical means and working bodies, identification of their advantages and disadvantages, creation of a theory of the technological process, development of criteria for optimizing parameters, search for new engineering calculation methods. To describe the processes of functioning of technical systems in most cases, methods of empirical and analytical modeling are used. The purpose of the work is to further search for a methodology for developing mechanical and technological models that describe the functional process of separating impurities from root crops by cleaning transport and technological modules of root harvesting machines. The article presents an algorithm for developing a mechanical and technological model and conducting research on complex dynamic systems that implement step-by-step operations of separating structural components of impurities from root crops.*

Keywords: *model, root crops, impurities, algorithm, structural diagram, block diagram, purification transport and technological module, parameters.*

1. INTRODUCTION

Scientific research of complex technical and technological machine systems is implemented on the basis of the application of modern methods of cognition, among which one of the most effective is the modeling method, the use of which allows for a comprehensive study of objects, processes, phenomena, etc. by creating an appropriate analogue or model [1].

The introduced concepts («algorithm» – Al Khorezmi) and open scientific knowledge of the exact sciences (Newton I., Copernicus N., Euler L. Leonardo da Vinci, Leverrier U., etc.) provided a significant impetus to the development of methodological principles that regulated new studies of the world space and the emergence of the first structural elements of modeling in technical systems and devices (Zhukovsky M., Tesla V., Tsiolkovsky K., etc.).

However, it was believed that mathematical modeling methods were not suitable for studying complex technical systems, while in the field of technology, the lack of objective mathematical models led both to the development of numerous partial engineering calculation methods that were prescription-based, and to the priority use of full-scale experiments, which, in the absence of other research options, led to subjective solutions [2].

The development and construction of an analog model of dynamic machine systems is carried out on the basis of the application of probability theory and the theory of random functions, mathematical statistics and computer mathematical programming, etc., while the final product of the study is the development and analysis of differential and integral equations that functionally describe the relationships and phenomena of the object of study or technological system itself [3].

In English, there are two terms for the concept of «modeling»: «modeling» and «simulation». The first means modeling based mainly on theoretical propositions, and the second means reproducing, imitating the real or approximate state of the system based on the analysis of its behavior (simulation modeling) [4, 5].

2. METHODS

Separately, in the methodology of mathematical modeling, one can distinguish mechanical and mathematical models that describe complex technological processes of the operation of basic units and technical systems of agricultural machines, including root harvesting machines, which are designed to harvest large, large-sized root crops of sugar and fodder beets, as well as chicory roots [6].

A separate important link in the study of the processes of operation of machines for harvesting root crops is the development and optimization of the parameters and modes of operation of the working bodies of the transport and cleaning systems of the root crop heap [7].

These models, as a rule, describe complex processes of mechanical energy transfer through the connections between the elements of the working bodies, the processed product and the environment.

When developing them, working hypotheses are put forward, which in a general context can be formulated as follows: mechanical and technological models of each technological operation to be synthesized must describe the energy state of the material being processed in the working space of the root harvester [8].

That is, such models must include a rheological model of the processing material or the environment with which the material interacts. In our case, these are rheological models of root crops and impurities – soil environment and tops, or a formalized version of the process in which the presence of foreign bodies in the dug up root crop pile is ignored. Such systems belong to dynamic systems, the feature of which is time dependence [9, 10].

Changes in a complex technical system over time are reflected in the state of the system itself, which is characterized by a quantity called the internal characteristic of the system. This characteristic determines the current (instantaneous) value of the output values, or parameters of the effect of the technical system [11, 12]. Therefore, such a structure of the mathematical model allows us to consider and analyze the interaction of working bodies with the processing material, without imposing restrictions on the change in the internal characteristics of the system over time.

The structure of the mechanical and mathematical model of the technological object of the study (Fig. 1) consists of a system of mathematical models that are necessary for vector optimization and are sufficiently connected to the database of this model. The database is based on a set of technological parameters T, which are in this case controlled or regulated [13]. The set of technological parameters T is formed from the agro-technological requirements for the process of the technical system and controls the internal values of the technological and mathematical model, which are divided into four blocks or sets of parameters [14]:

- a set of properties and characteristics of the processing material (the material of interaction of the working bodies with the processing product) M;
- a set of kinematic parameters of the process K;
- set of structural parameters of the technological object S;
- set of geometric parameters of the interaction space G.

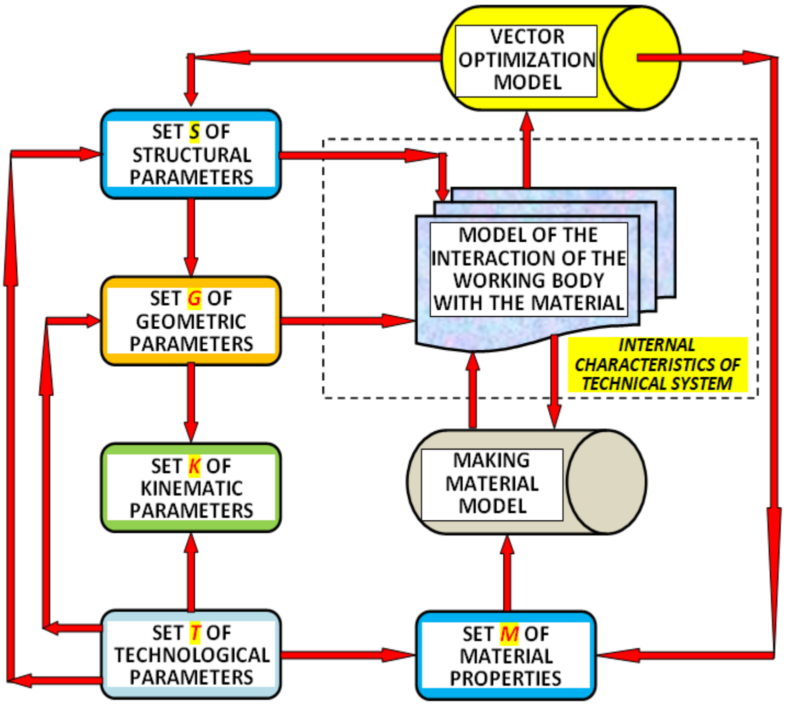

Figure 1. Structure of the mechanical-mathematical model of the technological object of research.

Thus, the following vectors are transferred to the mathematical model system as input data: a vector of parameters of the properties of the processing material from the set of permissible alternatives *M*; a vector of kinematic parameters of the process from the set of permissible alternatives *K*; a vector of structural parameters from the set of permissible alternatives *S*; a vector of geometric parameters of the interaction space from the set of permissible alternatives *G*.

The presented approach allows for scientific and methodological optimization of work processes and to justify rational parameters and modes of operation of the working bodies of the cleaning systems of root-harvesting machines depending on the accepted restrictions according to the target function or existing requirements.

Based on the analysis of the methodology for developing mathematical models that functionally describe the implementation of technological processes of complex technical systems, it can be stated that, taking into account the significant scientific achievements and the volume of known methods and principles for building structural and technological schemes and mathematical models, there are certain limitations and consequential reasons for their use for optimizing processes and parameters of working bodies for effective separation of impurities from root crops in difficult operating conditions of root harvesting machines [15–17].

The main shortcomings that do not allow to provide the necessary quality indicators of the technological process of separation of impurities from root crops when using known methodologies are as follows:

- the components of impurities are considered as a single continuous and continuous flow of material over a certain period of time, ignoring the fact that impurities are components of different structures in their composition, each of which significantly affects the process and has its own distinctive physical and mechanical characteristics, agrobiological and natural properties [18];
- the process of separation of impurities from root crops, or the number of separated impurities is described by dependencies that characterize the process of separation of a continuous flow along the entire length of the cleaning working body, or along the entire general path of movement of a continuous flow of root crops and impurities [19].

The deterministic model allows you to uniquely calculate the values of the output values from the values of the input parameters and control influences in real time.

In uncertain mathematical models, the values of factors and parameters are determined by random variables, which are given by the probable density of the random distribution of values [20].

The formalized essence, or value of the deterministic model, is that it allows the object of research, which interacts with the environment, to identify the general contours and properties of the formal technical system in a certain period of time, which will subsequently regulate the model system of the formal technical system of the *MFTS* (Fig. 2).

At the same time, the hypothetical process of developing a formal technical system, or the process of the researcher's interaction with part of the environment, is the result of the implementation of the set goal, or optimization criteria, which were formulated by the researcher on the basis of his interaction with the formalized environment, which formalizes the technical system of the *TS* (Fig. 2) and forms the functional model system of the *FM-S*.

Therefore, the development and improvement of structural and layout schemes of cleaning transport and technological modules (CTTM) of root harvesting machines and optimization of the parameters of their working bodies must be carried out taking into account the specific properties of the given subjective and objective environment [21].

This is especially important and relevant for the process of separating structural components of impurities from root crops and the development of designs of combined cleaning systems, the working bodies of which receive a significant amount of impurities:

- soil – up to 80%...90%, including adhered soil – up to 3%...5%;
- plant impurities – up to 10%...15%, residues of the tip on the heads of root crops – 5%...10% relative to the total impurities [22].

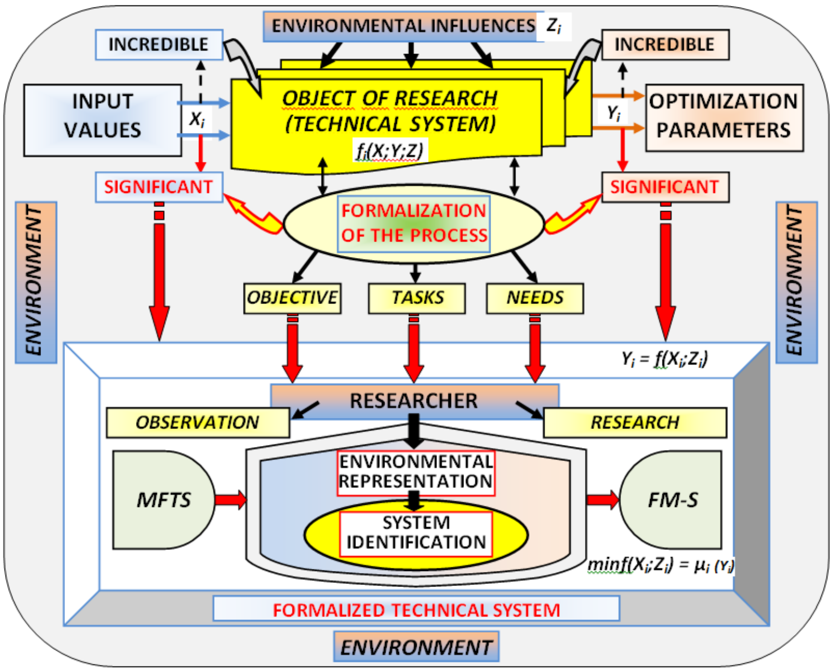

Figure 2. Scheme of the process of developing a functional system-model of a technical object of research (a complex dynamic technical system).

The art of modeling complex technical systems lies in the ability of a researcher or engineer to solve the following scientific problems in accordance with the set goal of the scientific researcher:

- analyze the existing current scientific and practical problem of the corresponding state of the art;
- isolate from it by abstraction its essential features or characteristics;
- select and, accordingly, modify the main formalized assumptions;

- develop and improve the developed preliminary mathematical or full-scale model of the technical system until it produces useful and positive results for practice.

The algorithm for developing CTTM root harvesting machines is inherently closely related to the technological complexity of cleaning the heap from structural components of impurities, which is dug out by the working bodies of the root crop digging module from the soil-root environment.

This is regulated by the technological necessity of ensuring significant separation of soil and plant impurities with different physical and mechanical properties (the supply of impurities is 4...8 kg/m2), which are in relation to root crops in free (loose soil, small 20...50 mm and large up to 100 mm lumps of soil, lost tops) and bound (sticky soil and tops residues on root crops) states [23, 24].

At the same time, total impurities can make up to 30%...40% of the number of dug root crops, depending on the operating conditions of the root harvesting machine [25, 26], which must be separated from the root crops, while ensuring the necessary quality indicators of work in accordance with agrotechnical requirements.

3. RESULTS AND DISCUSSION

The process of conducting scientific research on the mechanical and technological model of the CTTM root harvesting machine, or modeling the technological process of intensification of the separation of structural components of impurities from root crops is illustrated by the block diagram shown in Fig. 3.

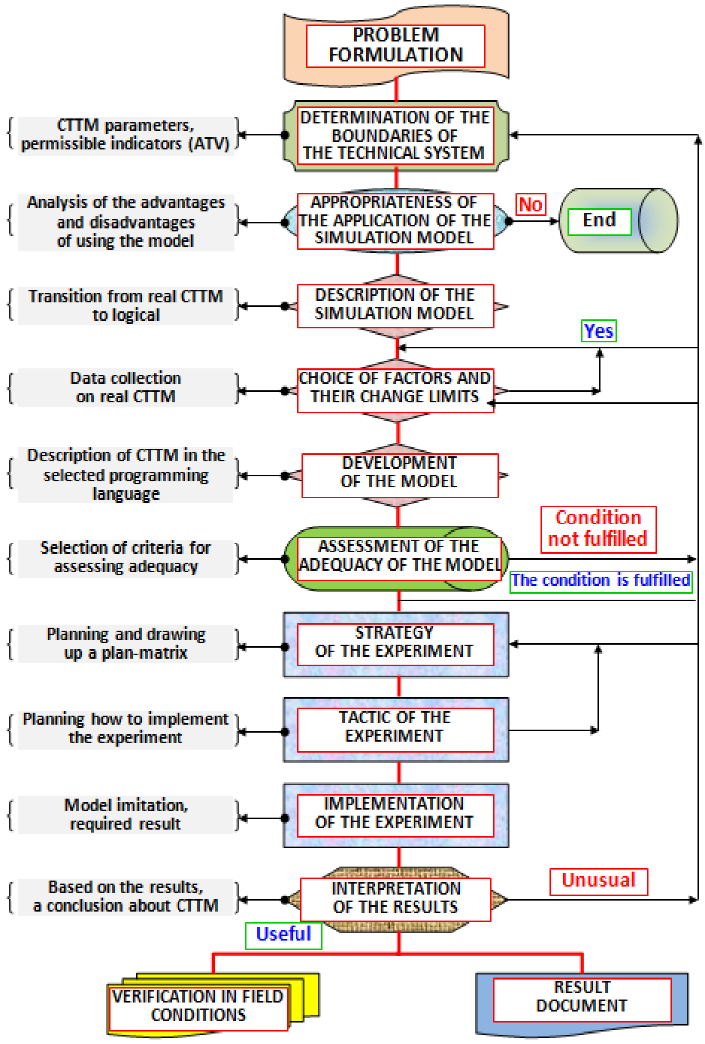

Figure 3. Flowchart of CTTM research stages.

The proposed algorithm for developing a technical system will provide a solution to the existing problem, or will allow a significant increase in the performance indicators of root harvesters by significantly reducing the supply of free soil and plant impurities, especially soil clods and bound soil and plant impurities, residues of the tip on the heads of root crops, and soil adhering to the lateral surface of the body of dug up root crops.

The process of constructive development of the mechanical and technological model of the CTTM of root harvesting machines, which is built in the form of intensified combined cleaning systems [27, 28], or technical systems for intensification of the functional process of separation of structural components of impurities from root crops is illustrated by the block diagrams shown in Fig. 4.

The development is based on the method and principle of modeling based on Bellman dynamic programming with the use of multi-stage optimization according to the method of Academician L. Pogoriliy with the addition of the stage of transport delay of moving the heap [29–31]. The optimal strategy of a dynamic model using multi-stage optimization has a certain property of the state (indicator, parameter, etc.) under which subsequent solutions of the dynamic model should have an optimal effect on the state of the first previous solution.

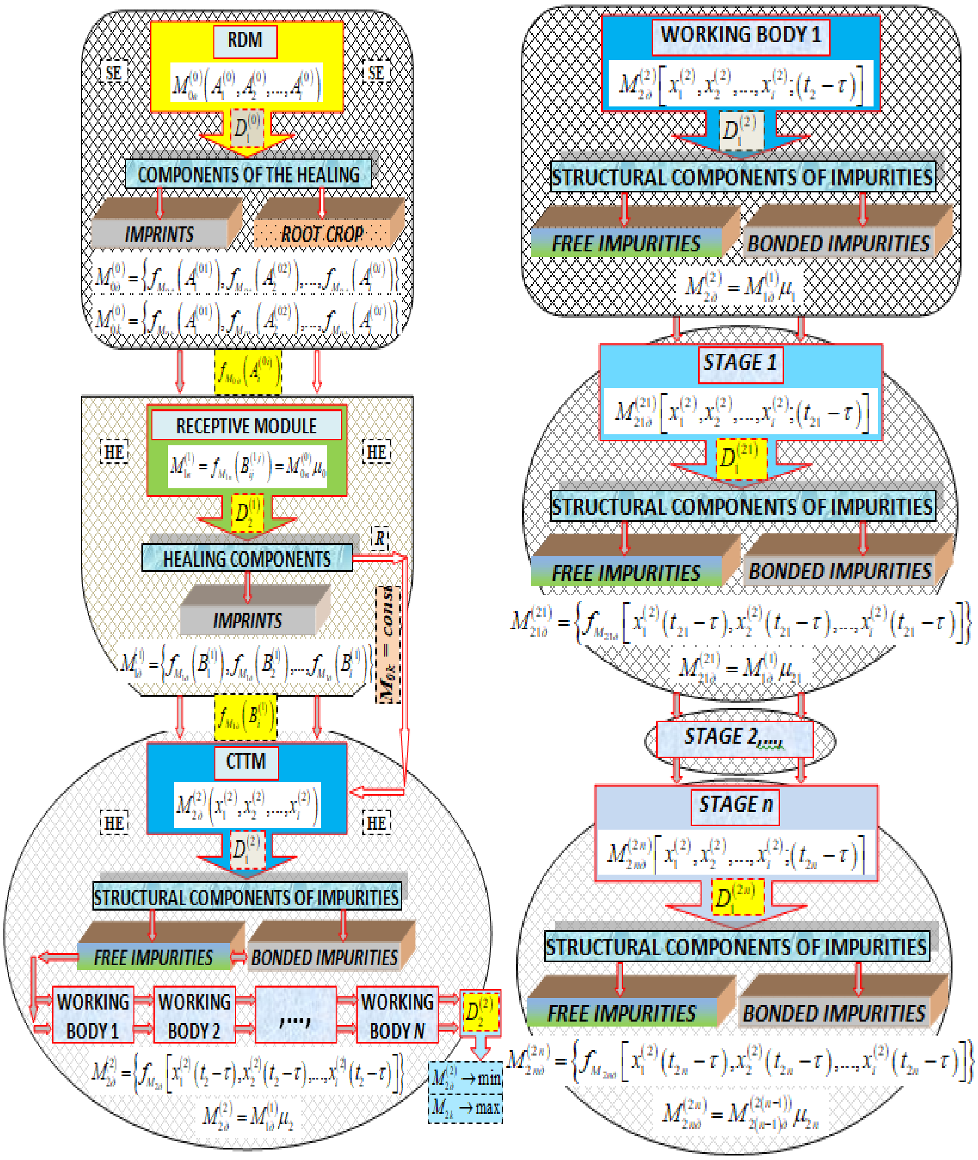

Figure 4. Flowchart of the development of a mechanical and technological model of the process of separating impurities from root crops: left – STTM in general; right – staged on one working body.

According to the Bellman principle, the optimal strategy of a dynamic model with the use of multi-stage optimization has a certain property of the state (indicator, parameter, etc.), that no matter what the initial state and the initial solution of the initial state are, the subsequent solutions of the dynamic model should constitute the optimal course of action with respect to the state obtained as a result of the first previous solution.

In the diagram of Fig. are marked:

- RDM – root crop digging module;
- SE – soil environment;
- HE – heap environment;
- R – root crops;
- $D_1^{(0)}$, $D_2^{(1)}$; $D_1^{(2)}$, $D_2^{(2)}$ – state of the research object (initial, specified optimum state);
- $A_1^{(0)}$, $A_2^{(0)}$, $A_i^{(0)}$ – respectively, structural parameters of the root crop digging process by the RDM working bodies;
- $B_1^{(1)}$, $B_2^{(1)}$, $B_i^{(1)}$ – respectively, structural parameters of the process of separation of impurities by the receiving module;
- $x_1^{(2)}$, $x_2^{(2)}$, $x_i^{(2)}$ – respectively, structural parameters of the process of separation of impurities from root crops by the working bodies of the CTTM;
- $M_{0в}^{(0)}$, $M_{0д}^{(0)}$, $M_{0k}^{(0)}$ – respectively, the mass of the heap, the mass of impurities and the mass of root crops fed to the working bodies of the receiving module;
- $M_{1в}^{(1)}$, $M_{1д}^{(1)}$ – respectively, the mass of the heap and the mass of impurities fed to the working bodies of the CTTM;
- $M_{2д}^{(2)}$ – the initial (set) mass of impurities;
- μ_1, μ_2 – the coefficients of separation of impurities;
- t_2, t_{21} – respectively, the total time of movement of impurities through the working bodies of the CTTM and through working body 1 at stage 1;
- τ – the time of transport delay of the movement of the constituent components of impurities and root crops.

In addition, the methodology of the principles of developing a mechanical and technological model of the CTTM operation process is based on our proposed method of step-by-step (multi-stage) separation of structural components of impurities from root crops by one working body (for example, working body 1, see Fig. 4, left), the essence of which is illustrated by the block diagram shown in Fig. 4 right. Increasing the efficiency of harvesting large-sized root crops largely depends on the further deepening and development of the general concept of building rational outlines of modern root harvesting machines and their structural components, or transport and technological modules, which ensure the sequence of execution of the main operations of harvesting root crops.

At the same time, the efficiency of the CTTM operation fundamentally depends on the technological aspects of the operation of the previous transport and technological modules, which perform the operations of harvesting tops and digging up root crops.

The principle of gradual (step-by-step) separation of structural components of impurities from root crops by one working body of the CTTM is implemented using the example of a scheme (Fig. 5), which is compiled according to the block diagrams, Fig. 4.

The principles of modeling technological processes and the quality of CTTM work are primarily regulated and largely depend on the technological supply of the components of the excavated heap to its working bodies. At the same time, the technological supply of the components of the excavated heap is functionally dependent on many factors: agrobiological and physical and mechanical properties of root crops and the surrounding soil environment; type and design of diggers; operating conditions of the root harvesting machine, etc. [32].

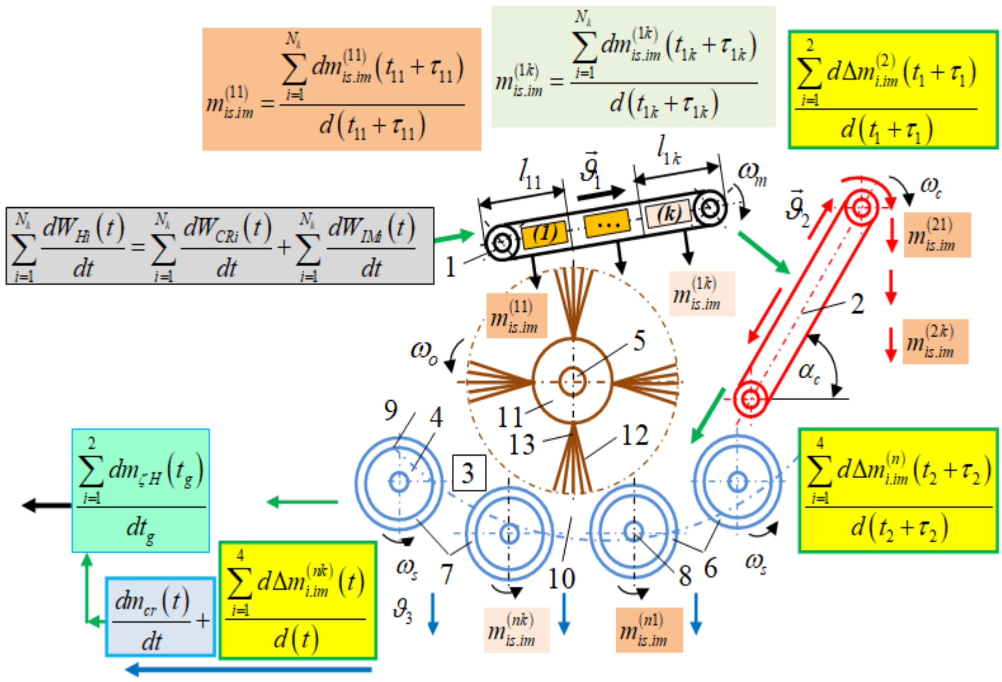

Figure 5. Model scheme for calculating the phased separation of impurities from root crops by working bodies of the CTTM: 1 – inclined conveyor; 2 – cleaning slide; 3 – combined working body; 4 – cleaning screws; 5 – cleaning shaft; 6, 7 – respectively, right and left system; 8 – screw shaft; 9 – lower branch of the ellipse; 10 – working channel chute; 11 – drum; 12 – elastic cleaning elements; 13– pile bundles.

In this case, it is necessary to develop a mathematical model that describes the step-by-step process of separating impurities from root crops on a separate j-th working body and a mathematical model of the process of cleaning root crops from impurities in the CTTM in general. We conventionally assume that the CTTM (Fig. 5) consists of j = 1, 2, …, n cleaning working bodies: the first (inclined conveyor 1), the second (cleaning slide 2), …, n-th cleaning working body (right and left system of screws 6 and 7, cleaning shaft 5), or j-th stages of cleaning.

Let the total mass quantity of structural and constituent components of the heap $\sum_{i=1}^{N_k} dW_{Hi}(t)/dt$ enter the working body 1 of the CTTM, where root crops are cleaned from impurities. To develop a step-by-step mathematical model, we conventionally divide the total length of the cleaning surface of each j-th working body of the CTTM into q = 1, 2, …, k sections (Fig. 5), where the process of separating impurities from the root crops takes place.

Let us denote the total mass quantity of separated impurities, both of soil and plant origin, on one separate j-th working body of the CTTM by $\sum_{i=1}^{2} dm_{is.im}^{(j)}\left(t_j+\tau_j\right)/d\left(t_j+\tau_j\right)$, where j = 1, 2,…,n; $t_j-\tau_j$ is the total time of finding the components of the heap on a separate j-th working body of the CTTM, or the time of implementation of the process of separating impurities from root crops, s. Here, the upper index in brackets (j) corresponds to the designation of the number of a specific working body from 1 to n, on which the process of cleaning root crops from impurities directly takes place.

Then

$$\sum_{i=1}^{2}\frac{dm_{is.im}^{(j)}\left(t_j+\tau_j\right)}{d\left(t_j+\tau_j\right)}=\sum_{i=1}^{2}\frac{dm_{is.s}^{(j)}\left(t_j+\tau_j\right)}{d\left(t_j+\tau_j\right)}+\sum_{i=1}^{2}\frac{dm_{is.p}^{(j)}\left(t_j+\tau_j\right)}{d\left(t_j+\tau_j\right)}, \qquad (1)$$

where $m_{is.s}^{(j)}$, $m_{is.p}^{(j)}$ is the total mass of separated, respectively, soil and plant impurities on the j-th working bodies of the CTTM, kg,
on

$$\begin{cases}\sum_{i=1}^{2}\frac{dm_{is.im}^{(1)}\left(t_1+\tau_1\right)}{d\left(t_1+\tau_1\right)}=\sum_{i=1}^{2}\frac{dm_{s.s}^{(1)}\left(t_1+\tau_1\right)}{d\left(t_1+\tau_1\right)}+\sum_{i=1}^{2}\frac{dm_{s.p}^{(1)}\left(t_1+\tau_1\right)}{d\left(t_1+\tau_1\right)}=\\ =\frac{dm_{s.fs}^{(1)}\left(t_1+\tau_1\right)}{d\left(t_1+\tau_1\right)}+\frac{dm_{s.fp}^{(1)}\left(t_1+\tau_1\right)}{d\left(t_1+\tau_1\right)}+\frac{dm_{s.ss}^{(1)}\left(t_1+\tau_1\right)}{d\left(t_1+\tau_1\right)}+\frac{dm_{s.rt}^{(1)}\left(t_1+\tau_1\right)}{d\left(t_1+\tau_1\right)};\\ \sum_{i=1}^{2}\frac{dm_{is.im}^{(2)}\left(t_2+\tau_2\right)}{d\left(t_2+\tau_2\right)}=\sum_{i=1}^{2}\frac{dm_{s.s}^{(2)}\left(t_2+\tau_2\right)}{d\left(t_2+\tau_2\right)}+\sum_{i=1}^{2}\frac{dm_{s.p}^{(2)}\left(t_2+\tau_2\right)}{d\left(t_2+\tau_2\right)}=\\ =\frac{dm_{s.fs}^{(2)}\left(t_2+\tau_2\right)}{d\left(t_2+\tau_2\right)}+\frac{dm_{s.fp}^{(2)}\left(t_2+\tau_2\right)}{d\left(t_2+\tau_2\right)}+\frac{dm_{s.ss}^{(2)}\left(t_2+\tau_2\right)}{d\left(t_2+\tau_2\right)}+\frac{dm_{s.rt}^{(2)}\left(t_2+\tau_2\right)}{d\left(t_2+\tau_2\right)};\\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots;\\ \sum_{i=1}^{2}\frac{dm_{is.im}^{(n)}\left(t_n+\tau_n\right)}{d\left(t_n+\tau_n\right)}=\sum_{i=1}^{2}\frac{dm_{s.s}^{(n)}\left(t_n+\tau_n\right)}{d\left(t_n+\tau_n\right)}+\sum_{i=1}^{2}\frac{dm_{s.p}^{(n)}\left(t_n+\tau_n\right)}{d\left(t_n+\tau_n\right)}=\\ =\frac{dm_{s.fs}^{(n)}\left(t_n+\tau_n\right)}{d\left(t_n+\tau_n\right)}+\frac{dm_{s.fp}^{(n)}\left(t_n+\tau_n\right)}{d\left(t_n+\tau_n\right)}+\frac{dm_{s.ss}^{(n)}\left(t_n+\tau_n\right)}{d\left(t_n+\tau_n\right)}+\frac{dm_{s.rt}^{(n)}\left(t_n+\tau_n\right)}{d\left(t_n+\tau_n\right)}\end{cases}, \qquad (2)$$

where $m_{s.s}^{(1)},m_{s.s}^{(2)},...,m_{s.s}^{(n)}$, $m_{s.p}^{(1)},m_{s.p}^{(2)},...,m_{s.p}^{(n)}$; $m_{s.fs}^{(1)},m_{s.fs}^{(2)},...,m_{s.fs}^{(n)}$, $m_{s.fp}^{(1)},m_{s.fp}^{(2)},...,m_{s.fp}^{(n)}$; $m_{s.ss}^{(1)},m_{s.ss}^{(2)},...,m_{s.ss}^{(n)}$, $m_{s.rt}^{(1)},m_{s.rt}^{(2)},...,m_{s.rt}^{(n)}$ – respectively, the total mass of separated: soil and plant impurities; free soil and plant impurities; separated stuck soil; of separated residues of tops on the heads of root crops on the 1st, 2nd, …, n-th working body of the STTM for a certain period of time, kg;
$(t_1+\tau_1),(t_2+\tau_2),...,(t_n+\tau_n)$ – total time of finding the components of the heap on the 1st, 2nd, …, n-th working body of the STTM, s.

The total number of separated components of free impurities by the inclined conveyor 1 of the STTM (Fig. 5), which are separated on each k-th section of the inclined conveyor taking into account the first equation (2) will be

$$\sum_{i=1}^{2}\frac{dm_{is.im}^{(1)}\left(t_1+\tau_1\right)}{d\left(t_1+\tau_1\right)}=\frac{dm_{s.fs}^{(11)}\left(t_{11}+\tau_{11}\right)}{d\left(t_{11}+\tau_{11}\right)}+\frac{dm_{s.fs}^{(12)}\left(t_{12}-\tau_{12}\right)}{d\left(t_{12}-\tau_{12}\right)}+,...,+\frac{dm_{s.fs}^{(1k)}\left(t_{1k}+\tau_{1k}\right)}{d\left(t_{1k}+\tau_{1k}\right)}+ \\ +\frac{dm_{s.fp}^{(11)}\left(t_{11}+\tau_{11}\right)}{d\left(t_{11}+\tau_{11}\right)}+\frac{dm_{s.fp}^{(12)}\left(t_{12}+\tau_{12}\right)}{d\left(t_{12}+\tau_{12}\right)}+,...,+\frac{dm_{s.fp}^{(1k)}\left(t_{1k}+\tau_{1k}\right)}{d\left(t_{1k}+\tau_{1k}\right)}. \qquad (3)$$

The total mass number of structural components of impurities that enter, for example, the working surface of the first section of the cleaning slide 2 (Fig. 5) and which we will denote by $\sum_{i=1}^{2}dm_{i.im}^{(12)}\left(t_1+\tau_1\right)/d\left(t_1+\tau_1\right)$ is equal to

$$\sum_{i=1}^{2}\frac{dm_{i.im}^{(12)}\left(t_1+\tau_1\right)}{d\left(t_1+\tau_1\right)}=\sum_{i=1}^{2}\frac{dm_{i.im}^{(1)}\left(t+\tau\right)}{d\left(t+\tau\right)}-\sum_{i=1}^{2}\frac{dm_{is.im}^{(1)}\left(t_1+\tau_1\right)}{d\left(t_1+\tau_1\right)}=\sum_{i=1}^{2}\frac{d\Delta m_{i.im}^{(2)}\left(t_1+\tau_1\right)}{d\left(t_1+\tau_1\right)}, \tag{4}$$

where $\sum_{i=1}^{2}dm_{i.im}^{(1)}\left(t+\tau\right)/d\left(t+\tau\right)$, $\sum_{i=1}^{2}dm_{is.im}^{(1)}\left(t_1+\tau_1\right)/d\left(t_1+\tau_1\right)$ – respectively, the total mass number of structural components of impurities that enter the inclined conveyor and the number of impurities that was separated on each k-th section of the inclined conveyor, kg.

Accordingly, by analogy with the notation of formula (3) and formula (4), it can be stated that:

- the total number of separated structural components of free impurities by the cleaning slide 2 (Fig. 5), which are separated at each q-th section of the cleaning slide taking into account the second Eq. (3) will be

$$\sum_{i=1}^{2}\frac{dm_{is.im}^{(2)}\left(t_2+\tau_2\right)}{d\left(t_2+\tau_2\right)}=\frac{dm_{s.fs}^{(21)}\left(t_{21}+\tau_{21}\right)}{d\left(t_{21}+\tau_{21}\right)}+\frac{dm_{s.fs}^{(22)}\left(t_{22}+\tau_{22}\right)}{d\left(t_{22}+\tau_{22}\right)}+,\ldots,+\frac{dm_{s.fs}^{(2k)}\left(t_{2k}+\tau_{2k}\right)}{d\left(t_{2k}+\tau_{2k}\right)}+$$
$$+\frac{dm_{s.fp}^{(21)}\left(t_{21}+\tau_{21}\right)}{d\left(t_{21}+\tau_{21}\right)}+\frac{dm_{s.fp}^{(22)}\left(t_{22}+\tau_{22}\right)}{d\left(t_{22}+\tau_{22}\right)}+,\ldots,+\frac{dm_{s.fp}^{(2k)}\left(t_{2k}+\tau_{2k}\right)}{d\left(t_{2k}+\tau_{2k}\right)}; \tag{5}$$

- the total mass number of structural components of impurities, which enters, for example, the first Eq. (1) section of the right 6 and left 7 screw systems will be

$$\sum_{i=1}^{2}\frac{dm_{i.im}^{(n)}\left(t_{n-1}+\tau_{n-1}\right)}{d\left(t_{n-1}+\tau_{n-1}\right)}=\sum_{i=1}^{2}\frac{dm_{i.im}^{(2)}\left(t_1+\tau_1\right)}{d\left(t_1+\tau_1\right)}-\sum_{i=1}^{2}\frac{dm_{is.im}^{(2)}\left(t_2+\tau_2\right)}{d\left(t_2+\tau_2\right)}-$$
$$-,\ldots,-\sum_{i=1}^{2}\frac{dm_{is.im}^{(2)}\left(t_{n-1}+\tau_{n-1}\right)}{d\left(t_{n-1}+\tau_{n-1}\right)}=\sum_{i=1}^{2}\frac{d\Delta m_{i.im}^{(n)}\left(t_{n-1}+\tau_{n-1}\right)}{d\left(t_{n-1}+\tau_{n-1}\right)}; \tag{6}$$

- the total number of separated structural components of free and bound impurities by the screw systems and elastic cleaning elements 12 of the cleaning shaft 5, which are separated at each q-th section of the right 6 and left 7 screw systems 4 of the combined cleaner 3 taking into account the n-th equation (3), will be

$$\sum_{i=1}^{2}\frac{dm_{is.im}^{(n)}\left(t_n+\tau_n\right)}{d\left(t_n+\tau_n\right)}=\frac{dm_{s.fs}^{(n1)}\left(t_{n1}+\tau_{n1}\right)}{d\left(t_{n1}+\tau_{n1}\right)}+\frac{dm_{s.fs}^{(n2)}\left(t_{n2}+\tau_{n2}\right)}{d\left(t_{n2}+\tau_{n2}\right)}+,\ldots,+\frac{dm_{s.fs}^{(nk)}\left(t_{nk}+\tau_{nk}\right)}{d\left(t_{nk}+\tau_{nk}\right)}+\frac{dm_{s.fp}^{(n1)}\left(t_{n1}+\tau_{n1}\right)}{d\left(t_{n1}+\tau_{n1}\right)}+$$
$$+\frac{dm_{s.fp}^{(n2)}\left(t_{n2}+\tau_{n2}\right)}{d\left(t_{n2}+\tau_{n2}\right)}+,\ldots,+\frac{dm_{s.fp}^{(nk)}\left(t_{nk}+\tau_{nk}\right)}{d\left(t_{nk}+\tau_{nk}\right)}+\frac{dm_{s.ss}^{(n1)}\left(t_{n1}+\tau_{n1}\right)}{d\left(t_{n1}+\tau_{n1}\right)}+\frac{dm_{s.ss}^{(n2)}\left(t_{n2}+\tau_{n2}\right)}{d\left(t_{n2}+\tau_{n2}\right)}+$$
$$+,\ldots,+\frac{dm_{s.ss}^{(nk)}\left(t_{nk}+\tau_{nk}\right)}{d\left(t_{nk}+\tau_{nk}\right)}+\frac{dm_{s.rt}^{(n1)}\left(t_{n1}+\tau_{n1}\right)}{d\left(t_{n1}+\tau_{n1}\right)}+\frac{dm_{s.rt}^{(n2)}\left(t_{n2}+\tau_{n2}\right)}{d\left(t_{n2}+\tau_{n2}\right)}+,\ldots,+\frac{dm_{s.rt}^{(nk)}\left(t_{nk}+\tau_{nk}\right)}{d\left(t_{nk}+\tau_{nk}\right)}. \tag{7}$$

Accordingly, the total mass quantity of the heap, which will be supplied to the following transport and technological systems of the root harvesting machine, taking into account Eq. (3) and Eq. (5), (6), (7), is determined by the formula

$$\sum_{i=1}^{2}\frac{dm_{i.st}\left(t_{t.t}\right)}{t_{t.t}}=\frac{dm_{\kappa}\left(t\right)}{dt}+\sum_{i=1}^{2}\frac{dm_{i.im}\left(t\right)}{dt}-\left(\sum_{i=1}^{2}\frac{dm_{is.im}^{(1)}\left(t_1+\tau_1\right)}{d\left(t_1+\tau_1\right)}+\sum_{i=1}^{3}\frac{dm_{is.im}^{(2)}\left(t_2+\tau_2\right)}{d\left(t_2+\tau_2\right)}+,\dots,+\sum_{i=1}^{4}\frac{dm_{is.im}^{(n)}\left(t_n+\tau_n\right)}{d\left(t_n+\tau_n\right)}\right). \quad (8)$$

The obtained differential equations are mathematical models, which in general form at the theoretical level allow functionally describing the step-by-step process of separation of structural components of impurities from root crops depending on the total time t_{jq} of finding the constituent structural components of impurities at each q-th section of each j-th working body of complex dynamic transport and cleaning systems of the heap of root harvesters.

4. CONCLUSIONS

1. In the existing mathematical models that describe the functional dependencies of the processes of separation of impurities (soil and plant) are single-stage, or generalizing. On their basis, the consideration of the process of separation of impurities from root crops is considered in the context of a single and simultaneous course of the process along the entire path of movement of root crop components and impurities along the surfaces of one working body of the purification system as a whole.
2. A method for developing a mathematical model (5)–(8) is proposed, which functionally describes the step-by-step process of separating structural components of impurities from root crops depending on the time of finding the constituent structural components of impurities at each *q*-th section of each *j*-th working body of complex dynamic transport and cleaning systems of the heap of root harvesting machines.
3. The presented approach allows for scientific and methodological optimization of work processes and justification of rational parameters and modes of operation of working bodies of technological and transport systems of root harvesting machines depending on the adopted restrictions in accordance with the target function or existing requirements.

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REFERENCES

[1] Volokha M. P. (2013) Principles of modeling technological processes of sugar beet production. Electronic edition. Problems of ecological biotechnology. NUBiP of Ukraine, no. 2. (In Ukrainian). Access mode: http://ecobio.nau. edu.ua/index.php/ecobiotech/article/view/5503/6215.

[2] Manjula E. V. P. J., Hiromi W. K. Chandana A. B., Melaaen M. C. (2017) A review of CFD modelling studies on pneumatic conveying and challenges in modelling offshore drill cuttings transport. Powder Technology, no. 305, pp. 782–793. Doi: 10.1016/j.powtec.2016.10.026.

[3] Adamchuk V. V., Baranov G. L., Baranovsky O. S. (2004). Modern trends in the development of agricultural technology. K.: Agrarna nauka, 396 p. (In Ukrainian).

[4] Antille D. L., Peets S., Galambošová J., Botta G. F., Rataj V., Macak M., Tullberg J. N., Chamen W. C. T., White D. R., Misiewicz P. A., Hargreaves P. R., Bienvenido J. F. & Godwin R. J. (2019) Review: Soil compaction and controlled traffic farming in arable and grass cropping systems. Agronomy Research,vol. 17 (3), pp. 653–682. Doi: 10.15159/AR.19.133.

[5] Baranovsky V. M. (2014). Mathematical model of intensification of the process of separation of impurities from root crops. Scientific Bulletin of NUBiPU. Series "Technology and Energy of the Agricultural Complex". K., issue 196, part 1, pp. 196–204. (In Ukrainian).

[6] Hevko R. B., Tkachenko I. G., Hlado Y. B., Sinii S. V., Gandziuk M. O., Trohaniak O. M. (2019). Mathematical model of a root harvester after-cleaning system. Bulletin of Karaganda University, no. 4 (96), pp. 81–89. Doi: 10.31489/2019M4/81-89.

[7] Kozachenko O., Pahuchiy A. (2019). Modeling of interaction with plants linseed occupancy drum TEKA. An International quarterly journal on motorization, Vehicle operation, energy efficiency and mechanical engineering, vol. 19, no. 1, pp. 59–64.

[8] Baranovsky V. M., Potapenko M. V. (2017). Theoretical analysis of the technological feed of lifted root crops. INMATEH – Agricultural Engineering, vol. 51 (1), pp. 29–38.

[9] Volokha M. P. (2020). Modeling of mechanized technological processes of growing and harvesting sugar beets: dissertation for the degree of Doctor of Technical Sciences: 05.05.11 – machines and means of mechanization of agricultural production. Kyiv, 343 p. (in Ukrainian).

[10] Mawla H. A., Yehia I., El Lithy A. M., Faisal A. (2012) An appropriate sugar-beet harvesting mechanization for big-scale project. MISR J. Agric. Eng, vol. 29, pp. 1213–1238.

[11] Baranovsky V. M., Pidgursky M. I., Pankiv M. R. (2014) Methodological and structural-technological aspects of the development of adapted root harvesting machines. Bulletin of the Ternopil National Technical University, no. 2 (74), pp. 106–113.

[12] Kozachenko O. V., Sedykh K. V., Shkregal O. M. (2020) Mathematical modeling of the stability of the mechanical system of the discator. Machinery & Energetics, no. 12 (4), pp. 61–66.

[13] Herasymchuk H. A., Baranovsky V. M., Herasymchuk O. O., Pastushenko A. S. (2018) Analytical research results of the combined root digger. INMATEH-Agricultural Engineering, vol. 54, no. 1, pp. 63–72.

[14] Baranovsky V., Pankiv M., Kukhar O., Gurik O. (2022) Results of the experimental investigations of fodder beets harvesting technologies. Scientific journal of TNTU, vol. 105, no. 2, pp. 6–16. https://doi.org/10.33108/visnyk_tntu2022.02.016.

[15] Pankiv M., Pidhurskyi M., Pylypets M., Babii A., Burda M. (2021) Method of step-by-step development of a mathematical model of the process of separating impurities from root crops. Scientific Journal of TNTU, vol. 104, no. 4, pp. 74–86. https://doi.org/10.33108/visnyk_tntu2021.04.074.

[16] Baranovsky V. M., Pidgursky M. I., Pankiv M. R., Teslyuk V. V., Onyshchenko V. B. (2014). Fundamentals of developing adapted transport and technological systems of root harvesting machines: monograph. Publisher: Ternopil National University named after I. Pulyuy, 351 p. (In Ukrainian).

[17] Pankiv M. R., Tesluk V. V., Baranovskij V. M. (2016) Investigation of the structural model of adapted machine for harvesting root crops. Innovative solutions in modern science, no. 8 (8), pp. 70–80.

[18] Baranovsky V. M. (2013). Mechanical and technological foundations of the development of

adapted root harvesting machines: author's abstract of the dissertation for the degree of Doctor of Technical Sciences: 05.05.11 – machines and means of mechanization of agricultural production. Ternopil, 2013. 40 p. (In Ukrainian).

[19] Gerasymchuk G. A., Baranovsky V. M., Pankiv M. R., Ramsh V. Yu. (2011) Analysis of the development trends of working bodies for separation of root crop heaps. Scientific notes, issue 21, vol. 1, pp. 119–123. (In Ukrainian).

[20] Baranovskiy V., Pankiv M., Dubchak N. (2017) Experimental research of stripping the leaves from root crops. Acta Technologica Agriculturae, vol. 20, is. 3, pp. 69–73.

[21] Hevko R., Zalutskyi S., Tkachenko I., Lyashuk O. (2021) Design development and study of an elastic sectional screw operating tool. Acta Polytechnica, vol. 61 (5), pp. 624–632. Doi: 10.14311/AP.2021.61.0624.

[22] Baranovskiy V. M., Pankiv M. R., Skalsky O.Yu., Pastushenko A.S. Chicory root crops combined harvester. INMATEH – Agricultural Engineering. 2017. Vol. 53. No. 3/2017. P. 41–50.

[23] Baranovsky V., Pankiv M., Jobbagy J., Marynenko S., Komar R. (2023) Theoretical-experimental investigations of the second serve of the root crop pile components. Acta Technologica Agriculturae, vol. 26, issue 1, pp. 49–58, Doi: 10.2478/ata-2023-0007.

[24] Baranovsky V. M., Voytyuk D. G., Kropivko S. V. (2003) Forecasting the amount of soil on the surface of the root crop body. “MOTRO’L 03” IV, vol. 6, pp. 164–172. (In Ukrainian).

[25] Baranovsky V. M., Pankiv M. R., Dubchak N. A., Onyshchenko V. B. (2008) Mathematical models of the mass of adhered soil on fodder beet roots. Visnyk of Lviv National Agrarian University: Agroengineering Research, no. 12 (2), pp. 314–326. (In Ukrainian).

[26] Berezhenko E., Pankiv M., Jobbagy J., Berezhenko B. (2021) Experimental research of the module for gathering plant of chicory root. Scientific Journal of the Ternopil National Technical University, no. 1 (101), pp. 56–67. Doi: 10.33108/visnyk_tntu2021.01.056.

[27] Baranovsky V., Tesliuk V., Lukach V., Ikalchyk M., Kushnirenko A., Kulyk V. (2021) The results of root crop cleaner experimental research. Scientific Journal of the Ternopil National Technical University, no. 1 (101), pp. 47–55. https://doi.org/10.33108/visnyk_tntu2021.01.

[28] Mou X., Ou Y., Liu Q., Feng J., Li Y. (2014) Kinematic analysis and experiments of elastic dentations in process of sugarcane leaf sheath stripping. Agricultural Mechanics Report, Journal of Shandong Agricultural University, vol. 2, pp. 122–129.

[29] Pogoriliy L. V. (2004) Beet harvesting machines: history, design, forecasting. K.: Fenix, 231 p. (In Russian).

[30] Jones Morgan, Peet Matthew M. (2021). A generalization of Bellman’s equation with application to path planning, obstacle avoidance and invariant set estimation. Automatica. 127 p. Doi:10.1016/j.automatica.2021.109510.

[31] Prikner P., Kotek M., Jindra P. (2017) Field compaction capacity of agricultural tyres. Agron. Res., vol. 15, pp. 806–816.

[32] Pankiv M. (2019) Modeling of the technological functioning process transport and cleaning system of roots. Innovative solutions in modern science, no. 9 (36), pp. 50–60.